1. $f(x) = 3x^2 - 2\cos x \quad \Rightarrow \quad f'(x) = 6x - 2(-\sin x) = 6x + 2\sin x$

3. $f(x) = \sin x + \frac{1}{2}\cot x \quad \Rightarrow \quad f'(x) = \cos x - \frac{1}{2}\csc^2 x$

5. $y = \sec\theta\,\tan\theta \quad \Rightarrow \quad y' = \sec\theta\,(\sec^2\theta) + \tan\theta\,(\sec\theta\tan\theta) = \sec\theta\,(\sec^2\theta + \tan^2\theta)$. Using the identity $1 + \tan^2\theta = \sec^2\theta$, we can write alternative forms of the answer as $\sec\theta\,(1 + 2\tan^2\theta)$ or $\sec\theta\,(2\sec^2\theta - 1)$.

8. $f(t) = \dfrac{\cot t}{e^t} \quad \Rightarrow \quad f'(t) = \dfrac{e^t(-\csc^2 t) - (\cot t)e^t}{(e^t)^2} = \dfrac{e^t(-\csc^2 t - \cot t)}{(e^t)^2} = -\dfrac{\csc^2 t + \cot t}{e^t}$

10. $y = \dfrac{1 + \sin x}{x + \cos x} \quad \Rightarrow$

$$y' = \frac{(x + \cos x)(\cos x) - (1 + \sin x)(1 - \sin x)}{(x + \cos x)^2} = \frac{x\cos x + \cos^2 x - (1 - \sin^2 x)}{(x + \cos x)^2}$$

$$= \frac{x\cos x + \cos^2 x - (\cos^2 x)}{(x + \cos x)^2} = \frac{x\cos x}{(x + \cos x)^2}$$

16. $\dfrac{d}{dx}(\sec x) = \dfrac{d}{dx}\left(\dfrac{1}{\cos x}\right) = \dfrac{(\cos x)(0) - 1(-\sin x)}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x} = \dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x} = \sec x\,\tan x$

20. $y = e^x \cos x \quad \Rightarrow \quad y' = e^x(-\sin x) + (\cos x)e^x = e^x(\cos x - \sin x) \quad \Rightarrow$ the slope of the tangent line at $(0, 1)$ is $e^0(\cos 0 - \sin 0) = 1(1 - 0) = 1$ and an equation is $y - 1 = 1(x - 0)$ or $y = x + 1$.

30. (a) $g(x) = f(x)\sin x \quad \Rightarrow \quad g'(x) = f(x)\cos x + \sin x \cdot f'(x)$, so

$$g'(\tfrac{\pi}{3}) = f(\tfrac{\pi}{3})\cos\tfrac{\pi}{3} + \sin\tfrac{\pi}{3} \cdot f'(\tfrac{\pi}{3}) = 4 \cdot \tfrac{1}{2} + \tfrac{\sqrt{3}}{2} \cdot (-2) = 2 - \sqrt{3}$$

(b) $h(x) = \dfrac{\cos x}{f(x)} \quad \Rightarrow \quad h'(x) = \dfrac{f(x) \cdot (-\sin x) - \cos x \cdot f'(x)}{[f(x)]^2}$, so

$$h'(\tfrac{\pi}{3}) = \frac{f(\frac{\pi}{3}) \cdot (-\sin\frac{\pi}{3}) - \cos\frac{\pi}{3} \cdot f'(\frac{\pi}{3})}{[f(\frac{\pi}{3})]^2} = \frac{4\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)(-2)}{4^2} = \frac{-2\sqrt{3} + 1}{16} = \frac{1 - 2\sqrt{3}}{16}$$

1. Let $u = g(x) = 1 + 4x$ and $y = f(u) = \sqrt[3]{u}$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (\frac{1}{3}u^{-2/3})(4) = \dfrac{4}{3\sqrt[3]{(1 + 4x)^2}}$.

5. Let $u = g(x) = \sqrt{x}$ and $y = f(u) = e^u$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (e^u)\left(\frac{1}{2}x^{-1/2}\right) = e^{\sqrt{x}} \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$.

7. $F(x) = (x^4 + 3x^2 - 2)^5 \quad \Rightarrow \quad F'(x) = 5(x^4 + 3x^2 - 2)^4 \cdot \dfrac{d}{dx}\left(x^4 + 3x^2 - 2\right) = 5(x^4 + 3x^2 - 2)^4(4x^3 + 6x)$

[or $10x(x^4 + 3x^2 - 2)^4(2x^2 + 3)$]

12. $f(t) = \sqrt[3]{1 + \tan t} = (1 + \tan t)^{1/3} \quad \Rightarrow \quad f'(t) = \frac{1}{3}(1 + \tan t)^{-2/3}\sec^2 t = \dfrac{\sec^2 t}{3\sqrt[3]{(1 + \tan t)^2}}$

32. $y = \sin(\sin(\sin x)) \quad \Rightarrow \quad y' = \cos(\sin(\sin x))\,\dfrac{d}{dx}\,(\sin(\sin x)) = \cos(\sin(\sin x))\ \cos(\sin x)\ \cos x$

36. $y = 2^{3^{x^2}} \quad \Rightarrow \quad y' = 2^{3^{x^2}}(\ln 2)\,\dfrac{d}{dx}\left(3^{x^2}\right) = 2^{3^{x^2}}(\ln 2)3^{x^2}(\ln 3)(2x)$

38. $y = \cos^2 x = (\cos x)^2 \quad \Rightarrow \quad y' = 2\cos x(-\sin x) = -2\cos x \sin x \quad \Rightarrow$

$y'' = (-2\cos x)\cos x + \sin x(2\sin x) = -2\cos^2 x + 2\sin^2 x$

Note: Many other forms of the answers exist. For example, $y' = -\sin 2x$ and $y'' = -2\cos 2x$.

42. $y = \sqrt{1+x^3} = (1+x^3)^{1/2} \quad \Rightarrow \quad y' = \frac{1}{2}(1+x^3)^{-1/2}\cdot 3x^2 = \dfrac{3x^2}{2\sqrt{1+x^3}}$. At $(2,3)$, $y' = \dfrac{3\cdot 4}{2\sqrt{9}} = 2$, and an equation of the tangent line is $y - 3 = 2(x-2)$, or $y = 2x - 1$.

52. $h(x) = \sqrt{4+3f(x)} \quad \Rightarrow \quad h'(x) = \frac{1}{2}(4+3f(x))^{-1/2}\cdot 3f'(x)$, so

$h'(1) = \frac{1}{2}(4+3f(1))^{-1/2}\cdot 3f'(1) = \frac{1}{2}(4+3\cdot 7)^{-1/2}\cdot 3\cdot 4 = \frac{6}{\sqrt{25}} = \frac{6}{5}$

56. (a) $h(x) = f(f(x)) \quad \Rightarrow \quad h'(x) = f'(f(x))f'(x)$. So $h'(2) = f'(f(2))f'(2) = f'(1)f'(2) \approx (-1)(-1) = 1$.

(b) $g(x) = f(x^2) \quad \Rightarrow \quad g'(x) = f'(x^2)\cdot\dfrac{d}{dx}\,(x^2) = f'(x^2)(2x)$. So $g'(2) = f'(2^2)(2\cdot 2) = 4f'(4) \approx 4(2) = 8$.

60. (a) $F(x) = f(x^\alpha) \quad \Rightarrow \quad F'(x) = f'(x^\alpha)\dfrac{d}{dx}\,(x^\alpha) = f'(x^\alpha)\alpha x^{\alpha-1}$

(b) $G(x) = [f(x)]^\alpha \quad \Rightarrow \quad G'(x) = \alpha\,[f(x)]^{\alpha-1}\,f'(x)$

80. $x = \cos\theta + \sin 2\theta, \quad y = \sin\theta + \cos 2\theta;\ \theta = 0.\quad \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{\cos\theta - 2\sin 2\theta}{-\sin\theta + 2\cos 2\theta}$. When $\theta = 0$, $(x,y) = (1,1)$ and $dy/dx = \frac{1}{2}$, so an equation of the tangent to the curve is $y - 1 = \frac{1}{2}(x-1)$, or $y = \frac{1}{2}x + \frac{1}{2}$.