1. (a) $\frac{d}{dx}(xy+2x+3x^2)=\frac{d}{dx}(4) \Rightarrow (x\cdot y'+y\cdot 1)+2+6x=0 \Rightarrow xy'=-y-2-6x \Rightarrow$
$y'=\frac{-y-2-6x}{x}$ or $y'=-6-\frac{y+2}{x}$.

(b) $xy+2x+3x^2=4 \Rightarrow xy=4-2x-3x^2 \Rightarrow y=\frac{4-2x-3x^2}{x}=\frac{4}{x}-2-3x$, so $y'=-\frac{4}{x^2}-3$.

(c) From part (a), $y'=\frac{-y-2-6x}{x}=\frac{-(4/x-2-3x)-2-6x}{x}=\frac{-4/x-3x}{x}=-\frac{4}{x^2}-3$.

3. $\frac{d}{dx}\left(x^3+y^3\right)=\frac{d}{dx}(1) \Rightarrow 3x^2+3y^2\cdot y'=0 \Rightarrow 3y^2\,y'=-3x^2 \Rightarrow y'=-\frac{x^2}{y^2}$

7. $\frac{d}{dx}\left[x^4(x+y)\right]=\frac{d}{dx}\left[y^2(3x-y)\right] \Rightarrow x^4(1+y')+(x+y)\cdot 4x^3=y^2(3-y')+(3x-y)\cdot 2y\,y' \Rightarrow$
$x^4+x^4\,y'+4x^4+4x^3y=3y^2-y^2\,y'+6xy\,y'-2y^2\,y' \Rightarrow x^4\,y'+3y^2\,y'-6xy\,y'=3y^2-5x^4-4x^3y \Rightarrow$
$(x^4+3y^2-6xy)\,y'=3y^2-5x^4-4x^3y \Rightarrow y'=\frac{3y^2-5x^4-4x^3y}{x^4+3y^2-6xy}$

10. $\frac{d}{dx}(1+x)=\frac{d}{dx}\left[\sin(xy^2)\right] \Rightarrow 1=[\cos(xy^2)](x\cdot 2y\,y'+y^2\cdot 1) \Rightarrow 1=2xy\cos(xy^2)\,y'+y^2\cos(xy^2) \Rightarrow$
$1-y^2\cos(xy^2)=2xy\cos(xy^2)\,y' \Rightarrow y'=\frac{1-y^2\cos(xy^2)}{2xy\cos(xy^2)}$

18. $\frac{d}{dx}\left[g(x)+x\sin g(x)\right]=\frac{d}{dx}(x^2) \Rightarrow g'(x)+x\cos g(x)\cdot g'(x)+\sin g(x)\cdot 1=2x$. If $x=0$, we have
$g'(0)+0+\sin g(0)=2(0) \Rightarrow g'(0)+\sin 0=0 \Rightarrow g'(0)+0=0 \Rightarrow g'(0)=0$.

26. $x^{2/3}+y^{2/3}=4 \Rightarrow \frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}y'=0 \Rightarrow \frac{1}{\sqrt[3]{x}}+\frac{y'}{\sqrt[3]{y}}=0 \Rightarrow y'=-\frac{\sqrt[3]{y}}{\sqrt[3]{x}}$. When $x=-3\sqrt{3}$
and $y=1$, we have $y'=-\frac{1}{\left(-3\sqrt{3}\right)^{1/3}}=-\frac{\left(-3\sqrt{3}\right)^{2/3}}{-3\sqrt{3}}=\frac{3}{3\sqrt{3}}=\frac{1}{\sqrt{3}}$, so an equation of the tangent line is
$y-1=\frac{1}{\sqrt{3}}\left(x+3\sqrt{3}\right)$ or $y=\frac{1}{\sqrt{3}}x+4$.

31. $9x^2+y^2=9 \Rightarrow 18x+2y\,y'=0 \Rightarrow 2y\,y'=-18x \Rightarrow y'=-9x/y \Rightarrow$
$y''=-9\left(\frac{y\cdot 1-x\cdot y'}{y^2}\right)=-9\left(\frac{y-x(-9x/y)}{y^2}\right)=-9\cdot\frac{y^2+9x^2}{y^3}=-9\cdot\frac{9}{y^3}$ [since x and y must satisfy the original equation, $9x^2+y^2=9$]. Thus, $y''=-81/y^3$.

56. $x^2 + 4y^2 = 5 \quad \Rightarrow \quad 2x + 4(2yy') = 0 \quad \Rightarrow \quad y' = -\dfrac{x}{4y}$. Now let h be the height of the lamp, and let (a, b) be the point of tangency of the line passing through the points $(3, h)$ and $(-5, 0)$. This line has slope $(h - 0)/[3 - (-5)] = \frac{1}{8}h$. But the slope of the tangent line through the point (a, b) can be expressed as $y' = -\dfrac{a}{4b}$, or as $\dfrac{b - 0}{a - (-5)} = \dfrac{b}{a + 5}$ [since the line passes through $(-5, 0)$ and (a, b)], so $-\dfrac{a}{4b} = \dfrac{b}{a + 5} \quad \Leftrightarrow \quad 4b^2 = -a^2 - 5a \quad \Leftrightarrow \quad a^2 + 4b^2 = -5a$. But $a^2 + 4b^2 = 5$ [since (a, b) is on the ellipse], so $5 = -5a \quad \Leftrightarrow \quad a = -1$. Then $4b^2 = -a^2 - 5a = -1 - 5(-1) = 4 \quad \Rightarrow \quad b = 1$, since the point is on the top half of the ellipse. So $\dfrac{h}{8} = \dfrac{b}{a + 5} = \dfrac{1}{-1 + 5} = \dfrac{1}{4} \quad \Rightarrow \quad h = 2$. So the lamp is located 2 units above the x-axis.

1. (a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ since $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\frac{\pi}{3}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

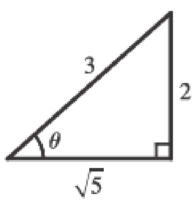
(b) $\cos^{-1}(-1) = \pi$ since $\cos\pi = -1$ and π is in $[0, \pi]$.

3. (a) $\arctan 1 = \frac{\pi}{4}$ since $\tan\frac{\pi}{4} = 1$ and $\frac{\pi}{4}$ is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(b) $\sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$ since $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $\frac{\pi}{4}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

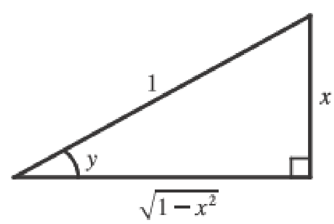
5. Let $\theta = \sin^{-1}\left(\frac{2}{3}\right)$.

Then $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right) = \tan\theta = \dfrac{2}{\sqrt{5}}$.



10. Let $y = \sin^{-1} x$. Then $\sin y = x$, so from the triangle we see that

$\tan(\sin^{-1} x) = \tan y = \dfrac{x}{\sqrt{1 - x^2}}$.



16. (a) Let $a = \sin^{-1} x$ and $b = \cos^{-1} x$. Then $\cos a = \sqrt{1-\sin^2 a} = \sqrt{1-x^2}$ since $\cos a \geq 0$ for $-\frac{\pi}{2} \leq a \leq \frac{\pi}{2}$. Similarly, $\sin b = \sqrt{1-x^2}$. So

$$\begin{aligned}\sin(\sin^{-1} x + \cos^{-1} x) &= \sin(a+b) = \sin a \cos b + \cos a \sin b = x \cdot x + \sqrt{1-x^2}\,\sqrt{1-x^2} \\ &= x^2 + (1-x^2) = 1\end{aligned}$$

But $-\frac{\pi}{2} \leq \sin^{-1} x + \cos^{-1} x \leq \frac{3\pi}{2}$, and so $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.

(b) We differentiate $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ with respect to x, and get

$$\frac{1}{\sqrt{1-x^2}} + \frac{d}{dx}(\cos^{-1} x) = 0 \quad \Rightarrow \quad \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}.$$

20. $F(\theta) = \arcsin\sqrt{\sin\theta} = \arcsin(\sin\theta)^{1/2} \quad \Rightarrow$

$$F'(\theta) = \frac{1}{\sqrt{1-\left(\sqrt{\sin\theta}\right)^2}} \cdot \frac{d}{d\theta}(\sin\theta)^{1/2} = \frac{1}{\sqrt{1-\sin\theta}} \cdot \frac{1}{2}(\sin\theta)^{-1/2} \cdot \cos\theta = \frac{\cos\theta}{2\sqrt{1-\sin\theta}\,\sqrt{\sin\theta}}$$

26. $y = \cos^{-1}(\sin^{-1} t) \quad \Rightarrow \quad y' = -\dfrac{1}{\sqrt{1-(\sin^{-1} t)^2}} \cdot \dfrac{d}{dt}\sin^{-1} t = -\dfrac{1}{\sqrt{1-(\sin^{-1} t)^2}} \cdot \dfrac{1}{\sqrt{1-t^2}}$

34. $y = 3\arccos\dfrac{x}{2} \quad \Rightarrow \quad y' = 3\left[-\dfrac{1}{\sqrt{1-(x/2)^2}}\right]\left(\dfrac{1}{2}\right)$, so at $(1,\pi)$, $y' = -\dfrac{3}{2\sqrt{1-\frac{1}{4}}} = -\sqrt{3}$. An equation of the tangent line is $y - \pi = -\sqrt{3}\,(x-1)$, or $y = -\sqrt{3}\,x + \pi + \sqrt{3}$.

2. $f(x) = x\ln x - x \quad \Rightarrow \quad f'(x) = x \cdot \dfrac{1}{x} + (\ln x) \cdot 1 - 1 = 1 + \ln x - 1 = \ln x$

5. $f(x) = \log_2(1-3x) \quad \Rightarrow \quad f'(x) = \dfrac{1}{(1-3x)\ln 2}\,\dfrac{d}{dx}(1-3x) = \dfrac{-3}{(1-3x)\,\ln 2}$ or $\dfrac{3}{(3x-1)\,\ln 2}$

11. $F(t) = \ln\dfrac{(2t+1)^3}{(3t-1)^4} = \ln(2t+1)^3 - \ln(3t-1)^4 = 3\ln(2t+1) - 4\ln(3t-1) \quad \Rightarrow$

$F'(t) = 3 \cdot \dfrac{1}{2t+1} \cdot 2 - 4 \cdot \dfrac{1}{3t-1} \cdot 3 = \dfrac{6}{2t+1} - \dfrac{12}{3t-1}$, or combined, $\dfrac{-6(t+3)}{(2t+1)(3t-1)}$.

19. $y = 2x\log_{10}\sqrt{x} = 2x\log_{10} x^{1/2} = 2x \cdot \frac{1}{2}\log_{10} x = x\log_{10} x \quad \Rightarrow \quad y' = x \cdot \dfrac{1}{x\ln 10} + \log_{10} x \cdot 1 = \dfrac{1}{\ln 10} + \log_{10} x$

Note: $\dfrac{1}{\ln 10} = \dfrac{\ln e}{\ln 10} = \log_{10} e$, so the answer could be written as $\dfrac{1}{\ln 10} + \log_{10} x = \log_{10} e + \log_{10} x = \log_{10} ex$.

24. $f(x) = \ln\ln\ln x \quad \Rightarrow \quad f'(x) = \dfrac{1}{\ln\ln x} \cdot \dfrac{1}{\ln x} \cdot \dfrac{1}{x}$.

$\text{Dom}(f) = \{x \mid \ln\ln x > 0\} = \{x \mid \ln x > 1\} = \{x \mid x > e\} = (e, \infty)$.

34. $y = \sqrt{x}\, e^{x^2} \left(x^2+1\right)^{10} \quad \Rightarrow \quad \ln y = \ln \sqrt{x} + \ln e^{x^2} + \ln(x^2+1)^{10} \quad \Rightarrow \quad \ln y = \frac{1}{2} \ln x + x^2 + 10 \ln(x^2+1) \quad \Rightarrow$

$$\frac{1}{y}\, y' = \frac{1}{2} \cdot \frac{1}{x} + 2x + 10 \cdot \frac{1}{x^2+1} \cdot 2x \quad \Rightarrow \quad y' = \sqrt{x}\, e^{x^2} (x^2+1)^{10} \left(\frac{1}{2x} + 2x + \frac{20x}{x^2+1} \right)$$

38. $y = x^{\cos x} \quad \Rightarrow \quad \ln y = \ln x^{\cos x} \quad \Rightarrow \quad \ln y = \cos x \ln x \quad \Rightarrow \quad \frac{1}{y}\, y' = \cos x \cdot \frac{1}{x} + \ln x \cdot (-\sin x) \quad \Rightarrow$

$$y' = y\left(\frac{\cos x}{x} - \ln x \sin x \right) \quad \Rightarrow \quad y' = x^{\cos x} \left(\frac{\cos x}{x} - \ln x \sin x \right)$$