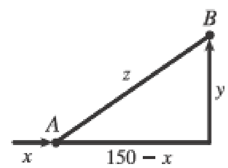
3. Let s denote the side of a square. The square's area A is given by $A = s^2$. Differentiating with respect to t gives us $\frac{dA}{dt} = 2s\,\frac{ds}{dt}$. When $A = 16$, $s = 4$. Substituting 4 for s and 6 for $\frac{ds}{dt}$ gives us $\frac{dA}{dt} = 2(4)(6) = 48\ \text{cm}^2/\text{s}$.

6. $V = \frac{4}{3}\pi r^3 \quad\Rightarrow\quad \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2\,\frac{dr}{dt} \quad\Rightarrow\quad \frac{dV}{dt} = 4\pi\left(\frac{1}{2}\cdot 80\right)^2(4) = 25{,}600\pi\ \text{mm}^3/\text{s}$.

12. (a) Given: at noon, ship A is 150 km west of ship B; ship A is sailing east at 35 km/h, and ship B is sailing north at 25 km/h. If we let t be time (in hours), x be the distance traveled by ship A (in km), and y be the distance traveled by ship B (in km), then we are given that $dx/dt = 35$ km/h and $dy/dt = 25$ km/h.

(b) Unknown: the rate at which the distance between the ships is changing at 4:00 PM. If we let z be the distance between the ships, then we want to find dz/dt when $t = 4$ h.

(c)

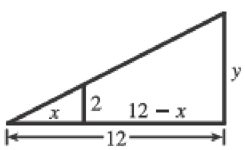


(d) $z^2 = (150 - x)^2 + y^2 \quad\Rightarrow\quad 2z\,\frac{dz}{dt} = 2(150 - x)\left(-\frac{dx}{dt}\right) + 2y\,\frac{dy}{dt}$

(e) At 4:00 PM, $x = 4(35) = 140$ and $y = 4(25) = 100 \quad\Rightarrow\quad z = \sqrt{(150 - 140)^2 + 100^2} = \sqrt{10{,}100}$.

So $\frac{dz}{dt} = \frac{1}{z}\left[(x - 150)\,\frac{dx}{dt} + y\,\frac{dy}{dt}\right] = \frac{-10(35) + 100(25)}{\sqrt{10{,}100}} = \frac{215}{\sqrt{101}} \approx 21.4$ km/h.

16.

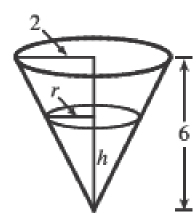


We are given that $\frac{dx}{dt} = 1.6$ m/s. By similar triangles, $\frac{y}{12} = \frac{2}{x} \quad\Rightarrow\quad y = \frac{24}{x} \quad\Rightarrow$ $\frac{dy}{dt} = -\frac{24}{x^2}\,\frac{dx}{dt} = -\frac{24}{x^2}(1.6)$. When $x = 8$, $\frac{dy}{dt} = -\frac{24(1.6)}{64} = -0.6$ m/s, so the shadow is decreasing at a rate of 0.6 m/s.

26. If $C =$ the rate at which water is pumped in, then $\frac{dV}{dt} = C - 10{,}000$, where $V = \frac{1}{3}\pi r^2 h$ is the volume at time t. By similar triangles, $\frac{r}{2} = \frac{h}{6} \quad\Rightarrow\quad r = \frac{1}{3}h \quad\Rightarrow$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h = \frac{\pi}{27}h^3 \quad\Rightarrow\quad \frac{dV}{dt} = \frac{\pi}{9}h^2\,\frac{dh}{dt}$. When $h = 200$ cm,

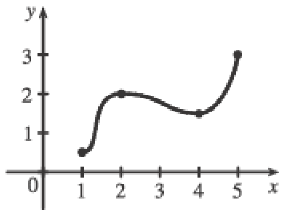
$\frac{dh}{dt} = 20$ cm/min, so $C - 10{,}000 = \frac{\pi}{9}(200)^2(20) \quad\Rightarrow\quad C = 10{,}000 + \frac{800{,}000}{9}\pi \approx 289{,}253\ \text{cm}^3/\text{min}$.



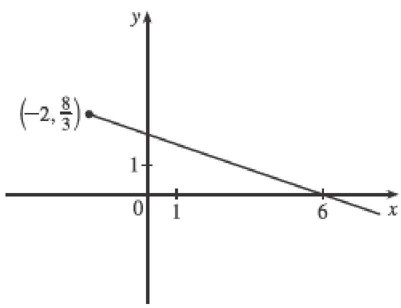
3. Absolute maximum at s, absolute minimum at r, local maximum at c, local minima at b and r, neither a maximum nor a minimum at a and d.

5. Absolute maximum value is $f(4) = 5$; there is no absolute minimum value; local maximum values are $f(4) = 5$ and $f(6) = 4$; local minimum values are $f(2) = 2$ and $f(1) = f(5) = 3$.

8. Absolute minimum at 1, absolute maximum at 5, local maximum at 2, local minimum at 4



16. $f(x) = 2 - \frac{1}{3}x$, $x \geq -2$. Absolute maximum $f(-2) = \frac{8}{3}$; no local maximum. No absolute or local minimum.



24. $f(x) = x^3 + 6x^2 - 15x \quad \Rightarrow \quad f'(x) = 3x^2 + 12x - 15 = 3(x^2 + 4x - 5) = 3(x+5)(x-1)$.

$f'(x) = 0 \quad \Rightarrow \quad x = -5, 1$. These are the only critical numbers.

28. $g(t) = |3t - 4| = \begin{cases} 3t - 4 & \text{if } 3t - 4 \geq 0 \\ -(3t-4) & \text{if } 3t - 4 < 0 \end{cases} = \begin{cases} 3t - 4 & \text{if } t \geq \frac{4}{3} \\ 4 - 3t & \text{if } t < \frac{4}{3} \end{cases}$

$g'(t) = \begin{cases} 3 & \text{if } t > \frac{4}{3} \\ -3 & \text{if } t < \frac{4}{3} \end{cases}$ and $g'(t)$ does not exist at $t = \frac{4}{3}$, so $t = \frac{4}{3}$ is a critical number.

30. $h(p) = \dfrac{p-1}{p^2+4} \quad \Rightarrow \quad h'(p) = \dfrac{(p^2+4)(1) - (p-1)(2p)}{(p^2+4)^2} = \dfrac{p^2 + 4 - 2p^2 + 2p}{(p^2+4)^2} = \dfrac{-p^2 + 2p + 4}{(p^2+4)^2}$.

$h'(p) = 0 \quad \Rightarrow \quad p = \dfrac{-2 \pm \sqrt{4 + 16}}{-2} = 1 \pm \sqrt{5}$. The critical numbers are $1 \pm \sqrt{5}$. [$h'(p)$ exists for all real numbers.]

34. $g(\theta) = 4\theta - \tan\theta \quad \Rightarrow \quad g'(\theta) = 4 - \sec^2\theta$. $\quad g'(\theta) = 0 \quad \Rightarrow \quad \sec^2\theta = 4 \quad \Rightarrow \quad \sec\theta = \pm 2 \quad \Rightarrow \quad \cos\theta = \pm\frac{1}{2} \quad \Rightarrow$

$\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi$, and $\frac{4\pi}{3} + 2n\pi$ are critical numbers.

Note: The values of θ that make $g'(\theta)$ undefined are not in the domain of g.

42. $f(x) = 5 + 54x - 2x^3$, $[0, 4]$. $\quad f'(x) = 54 - 6x^2 = 6(9 - x^2) = 6(3+x)(3-x) = 0 \quad \Leftrightarrow \quad x = -3, 3$. $\quad f(0) = 5$, $f(3) = 113$, and $f(4) = 93$. So $f(3) = 113$ is the absolute maximum value and $f(0) = 5$ is the absolute minimum value.

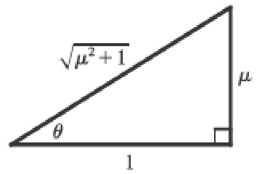
48. $f(x) = \dfrac{x^2-4}{x^2+4}$, $[-4,4]$. $f'(x) = \dfrac{(x^2+4)(2x)-(x^2-4)(2x)}{(x^2+4)^2} = \dfrac{16x}{(x^2+4)^2} = 0 \Leftrightarrow x = 0$. $f(\pm 4) = \frac{12}{20} = \frac{3}{5}$ and $f(0) = -1$. So $f(\pm 4) = \frac{3}{5}$ is the absolute maximum value and $f(0) = -1$ is the absolute minimum value.

50. $f(x) = x - \ln x$, $[\frac{1}{2}, 2]$. $f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$. $f'(x) = 0 \Rightarrow x = 1$. [Note that 0 is not in the domain of f.] $f(\frac{1}{2}) = \frac{1}{2} - \ln\frac{1}{2} \approx 1.19$, $f(1) = 1$, and $f(2) = 2 - \ln 2 \approx 1.31$. So $f(2) = 2 - \ln 2$ is the absolute maximum value and $f(1) = 1$ is the absolute minimum value.

62. $F = \dfrac{\mu W}{\mu\sin\theta + \cos\theta} \Rightarrow \dfrac{dF}{d\theta} = \dfrac{(\mu\sin\theta+\cos\theta)(0) - \mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta+\cos\theta)^2} = \dfrac{-\mu W(\mu\cos\theta-\sin\theta)}{(\mu\sin\theta+\cos\theta)^2}$.

So $\dfrac{dF}{d\theta} = 0 \Rightarrow \mu\cos\theta - \sin\theta = 0 \Rightarrow \mu = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$. Substituting $\tan\theta$ for μ in F gives us

$$F = \frac{(\tan\theta)W}{(\tan\theta)\sin\theta + \cos\theta} = \frac{W\tan\theta}{\dfrac{\sin^2\theta}{\cos\theta} + \cos\theta} = \frac{W\tan\theta\,\cos\theta}{\sin^2\theta + \cos^2\theta} = \frac{W\sin\theta}{1} = W\sin\theta.$$



If $\tan\theta = \mu$, then $\sin\theta = \dfrac{\mu}{\sqrt{\mu^2+1}}$ (see the figure), so $F = \dfrac{\mu}{\sqrt{\mu^2+1}}W$.

We compare this with the value of F at the endpoints: $F(0) = \mu W$ and $F(\frac{\pi}{2}) = W$.

Now because $\dfrac{\mu}{\sqrt{\mu^2+1}} \le 1$ and $\dfrac{\mu}{\sqrt{\mu^2+1}} \le \mu$, we have that $\dfrac{\mu}{\sqrt{\mu^2+1}}W$ is less than or equal to each of $F(0)$ and $F(\frac{\pi}{2})$.

Hence, $\dfrac{\mu}{\sqrt{\mu^2+1}}W$ is the absolute minimum value of $F(\theta)$, and it occurs when $\tan\theta = \mu$.