2. (a) f is concave upward on $(0, 2)$. (b) f is concave downward on $(2, 4)$ and $(4, 6)$.

(c) The point of inflection is $(2, 3)$.

8. (a) $f(x) = 4x^3 + 3x^2 - 6x + 1 \quad\Rightarrow\quad f'(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1) = 6(2x - 1)(x + 1)$. Thus, $f'(x) > 0 \quad\Leftrightarrow\quad x < -1$ or $x > \frac{1}{2}$ and $f'(x) < 0 \quad\Leftrightarrow\quad -1 < x < \frac{1}{2}$. So f is increasing on $(-\infty, -1)$ and $\left(\frac{1}{2}, \infty\right)$ and f is decreasing on $\left(-1, \frac{1}{2}\right)$.

(b) f changes from increasing to decreasing at $x = -1$ and from decreasing to increasing at $x = \frac{1}{2}$. Thus, $f(-1) = 6$ is a local maximum value and $f\left(\frac{1}{2}\right) = -\frac{3}{4}$ is a local minimum value.

(c) $f''(x) = 24x + 6 = 6(4x + 1)$. $\quad f''(x) > 0 \quad\Leftrightarrow\quad x > -\frac{1}{4}$ and $f''(x) < 0 \quad\Leftrightarrow\quad x < -\frac{1}{4}$. Thus, f is concave upward on $\left(-\frac{1}{4}, \infty\right)$ and concave downward on $\left(-\infty, -\frac{1}{4}\right)$. There is an inflection point at $\left(-\frac{1}{4}, f\left(-\frac{1}{4}\right)\right) = \left(-\frac{1}{4}, \frac{21}{8}\right)$.

16. (a) $f(x) = \sqrt{x}\, e^{-x} \quad\Rightarrow\quad f'(x) = \sqrt{x}\,(-e^{-x}) + e^{-x}\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}e^{-x}(-2x + 1) = \dfrac{1 - 2x}{2\sqrt{x}\, e^{x}}$.

$f'(x) > 0 \quad\Rightarrow\quad 1 - 2x > 0 \quad\Rightarrow\quad 2x < 1 \quad\Rightarrow\quad x < \frac{1}{2}$ and $f'(x) < 0 \quad\Rightarrow\quad x > \frac{1}{2}$. So f is increasing on $\left(0, \frac{1}{2}\right)$ and f is decreasing on $\left(\frac{1}{2}, \infty\right)$.

(b) f changes from increasing to decreasing at $x = \frac{1}{2}$. $\quad f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{2}}\, e^{-1/2} = \sqrt{\frac{1}{2}} \cdot \dfrac{1}{e^{1/2}} = 1/\sqrt{2e}$ [≈ 0.43].

Thus, $f\left(\frac{1}{2}\right) = 1/\sqrt{2e}$ is a local maximum value.

(c) $f'(x) = x^{-1/2}e^{-x}\left(-x + \frac{1}{2}\right) \quad\Rightarrow$

$$\begin{aligned} f''(x) &= x^{-1/2}e^{-x}\left(-x + \tfrac{1}{2}\right)' + x^{-1/2}\left(e^{-x}\right)'\left(-x + \tfrac{1}{2}\right) + (x^{-1/2})'e^{-x}\left(-x + \tfrac{1}{2}\right) \\ &= x^{-1/2}e^{-x}(-1) + x^{-1/2}(-e^{-x})\left(-x + \tfrac{1}{2}\right) + \left(-\tfrac{1}{2}x^{-3/2}\right)e^{-x}\left(-x + \tfrac{1}{2}\right) \\ &= x^{-3/2}e^{-x}\left[-x - x\left(-x + \tfrac{1}{2}\right) - \tfrac{1}{2}\left(-x + \tfrac{1}{2}\right)\right] = x^{-3/2}e^{-x}\left(x^2 - x - \tfrac{1}{4}\right) \end{aligned}$$

$f''(x) = 0 \quad\Rightarrow\quad x^2 - x - \frac{1}{4} = 0 \quad\Rightarrow\quad x = \dfrac{1 \pm \sqrt{1 + 1}}{2} = \frac{1}{2} \pm \frac{1}{2}\sqrt{2}$ [$\approx 1.21, -0.21$]. The domain of f is $[0, \infty)$, so we consider only $a = \frac{1}{2} + \frac{1}{2}\sqrt{2}$. $f''(x) > 0 \quad\Rightarrow\quad x > a$. Thus, f is concave upward on (a, ∞) and f is concave downward on $(0, a)$. There is a point of inflection at $(a, f(a)) \approx (1.21, 0.33)$.

18. $f(x) = \dfrac{x}{x^2+4} \quad \Rightarrow \quad f'(x) = \dfrac{(x^2+4)\cdot 1 - x(2x)}{(x^2+4)^2} = \dfrac{4-x^2}{(x^2+4)^2} = \dfrac{(2+x)(2-x)}{(x^2+4)^2}$.

First Derivative Test: $f'(x) > 0 \quad \Rightarrow \quad -2 < x < 2$ and $f'(x) < 0 \quad \Rightarrow \quad x > 2$ or $x < -2$. Since f' changes from positive to negative at $x = 2$, $f(2) = \frac{1}{4}$ is a local maximum value; and since f' changes from negative to positive at $x = -2$, $f(-2) = -\frac{1}{4}$ is a local minimum value.

Second Derivative Test:

$$f''(x) = \frac{(x^2+4)^2(-2x) - (4-x^2)\cdot 2(x^2+4)(2x)}{[(x^2+4)^2]^2} = \frac{-2x(x^2+4)\left[(x^2+4)+2(4-x^2)\right]}{(x^2+4)^4} = \frac{-2x(12-x^2)}{(x^2+4)^3}.$$

$f'(x) = 0 \quad \Leftrightarrow \quad x = \pm 2$. $f''(-2) = \frac{1}{16} > 0 \quad \Rightarrow \quad f(-2) = -\frac{1}{4}$ is a local minimum value.

$f''(2) = -\frac{1}{16} < 0 \quad \Rightarrow \quad f(2) = \frac{1}{4}$ is a local maximum value.

Preference: Since calculating the second derivative is fairly difficult, the First Derivative Test is easier to use for this function.

24. (a) $g(x) = 200 + 8x^3 + x^4 \quad \Rightarrow \quad g'(x) = 24x^2 + 4x^3 = 4x^2(6+x) = 0$ when $x = -6$ and when $x = 0$.

$g'(x) > 0 \quad \Leftrightarrow \quad x > -6 \quad [x \neq 0]$ and $g'(x) < 0 \quad \Leftrightarrow \quad x < -6$, so g is decreasing on $(-\infty, -6)$ and g is increasing on $(-6, \infty)$, with a horizontal tangent at $x = 0$.

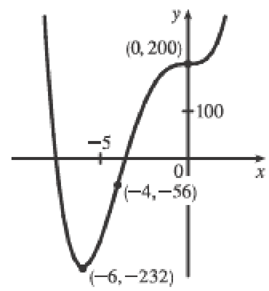
(b) $g(-6) = -232$ is a local minimum value.

There is no local maximum value.

(c) $g''(x) = 48x + 12x^2 = 12x(4+x) = 0$ when $x = -4$ and when $x = 0$.

$g''(x) > 0 \quad \Leftrightarrow \quad x < -4$ or $x > 0$ and $g''(x) < 0 \quad \Leftrightarrow \quad -4 < x < 0$, so g is CU on $(-\infty, -4)$ and $(0, \infty)$, and g is CD on $(-4, 0)$. There are inflection points at $(-4, -56)$ and $(0, 200)$.

(d)



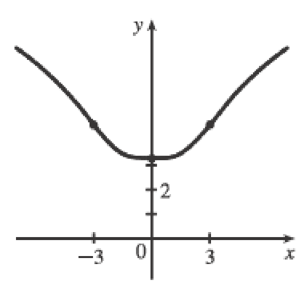
30. (a) $f(x) = \ln(x^4 + 27) \quad \Rightarrow \quad f'(x) = \dfrac{4x^3}{x^4+27}$. $f'(x) > 0$ if $x > 0$ and $f'(x) < 0$ if $x < 0$, so f is increasing on $(0, \infty)$ and f is decreasing on $(-\infty, 0)$.

(b) $f(0) = \ln 27 \approx 3.3$ is a local minimum value.

(c)
$$f''(x) = \frac{(x^4+27)(12x^2) - 4x^3(4x^3)}{(x^4+27)^2} = \frac{4x^2\left[3(x^4+27) - 4x^4\right]}{(x^4+27)^2}$$
$$= \frac{4x^2(81-x^4)}{(x^4+27)^2} = \frac{-4x^2(x^2+9)(x+3)(x-3)}{(x^4+27)^2}$$

$f''(x) > 0$ if $-3 < x < 0$ and $0 < x < 3$, and $f''(x) < 0$ if $x < -3$ or $x > 3$. Thus, f is concave upward on $(-3, 0)$ and $(0, 3)$ [hence on $(-3, 3)$] and f is concave downward on $(-\infty, -3)$ and $(3, \infty)$. There are inflection points at $(\pm 3, \ln 108) \approx (\pm 3, 4.68)$.

(d)



38. $f(x) = \dfrac{e^x}{1+e^x}$ has domain $\mathbb{R}$.

(a) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{e^x/e^x}{(1+e^x)/e^x} = \lim\limits_{x\to\infty} \dfrac{1}{e^{-x}+1} = \dfrac{1}{0+1} = 1$, so $y=1$ is a HA.

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{e^x}{1+e^x} = \dfrac{0}{1+0} = 0$, so $y=0$ is a HA. No VA.

(b) $f'(x) = \dfrac{(1+e^x)e^x - e^x\cdot e^x}{(1+e^x)^2} = \dfrac{e^x}{(1+e^x)^2} > 0$ for all x. Thus, f is increasing on $\mathbb{R}$.

(c) There is no local maximum or minimum.

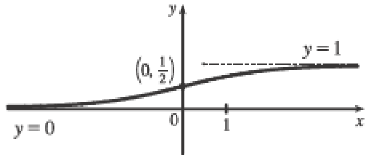
(d) $f''(x) = \dfrac{(1+e^x)^2 e^x - e^x\cdot 2(1+e^x)e^x}{[(1+e^x)^2]^2}$

$= \dfrac{e^x(1+e^x)[(1+e^x)-2e^x]}{(1+e^x)^4} = \dfrac{e^x(1-e^x)}{(1+e^x)^3}$

$f''(x) > 0 \quad\Leftrightarrow\quad 1-e^x > 0 \quad\Leftrightarrow\quad x<0$, so f is CU on $(-\infty,0)$ and CD on $(0,\infty)$.

There is an inflection point at $\left(0,\frac{1}{2}\right)$.

(e)



60. $f(x) = axe^{bx^2} \quad\Rightarrow\quad f'(x) = a\left[xe^{bx^2}\cdot 2bx + e^{bx^2}\cdot 1\right] = ae^{bx^2}(2bx^2+1)$. For $f(2)=1$ to be a maximum value, we must have $f'(2)=0$. $f(2)=1 \quad\Rightarrow\quad 1 = 2ae^{4b}$ and $f'(2)=0 \quad\Rightarrow\quad 0 = (8b+1)ae^{4b}$. So $8b+1=0$ $[a\neq 0]$ $\Rightarrow$ $b=-\frac{1}{8}$ and now $1 = 2ae^{-1/2} \quad\Rightarrow\quad a=\sqrt{e}/2$.

2. (a) $\lim\limits_{x\to a}[f(x)p(x)]$ is an indeterminate form of type $0\cdot\infty$.

(b) When x is near a, $p(x)$ is large and $h(x)$ is near 1, so $h(x)p(x)$ is large. Thus, $\lim\limits_{x\to a}[h(x)p(x)] = \infty$.

(c) When x is near a, $p(x)$ and $q(x)$ are both large, so $p(x)q(x)$ is large. Thus, $\lim\limits_{x\to a}[p(x)q(x)] = \infty$.

6. This limit has the form $\frac{0}{0}$. $\lim\limits_{x\to 1}\dfrac{x^a-1}{x^b-1} \overset{\text{H}}{=} \lim\limits_{x\to 1}\dfrac{ax^{a-1}}{bx^{b-1}} = \dfrac{a}{b}$

8. This limit has the form $\frac{0}{0}$. $\lim\limits_{x\to 0}\dfrac{\sin 4x}{\tan 5x} \overset{\text{H}}{=} \lim\limits_{x\to 0}\dfrac{4\cos 4x}{5\sec^2(5x)} = \dfrac{4(1)}{5(1)^2} = \dfrac{4}{5}$

14. This limit has the form $\frac{\infty}{\infty}$. $\lim\limits_{x\to\infty}\dfrac{(\ln x)^2}{x} \overset{\text{H}}{=} \lim\limits_{x\to\infty}\dfrac{2(\ln x)(1/x)}{1} = 2\lim\limits_{x\to\infty}\dfrac{\ln x}{x} \overset{\text{H}}{=} 2\lim\limits_{x\to\infty}\dfrac{1/x}{1} = 2(0) = 0$

16. This limit has the form $\frac{0}{0}$. $\lim\limits_{x\to 1}\dfrac{\ln x}{\sin \pi x} \overset{\text{H}}{=} \lim\limits_{x\to 1}\dfrac{1/x}{\pi\cos\pi x} = \dfrac{1}{\pi(-1)} = -\dfrac{1}{\pi}$

20. This limit has the form $\frac{0}{0}$.

$$\lim_{x\to 0}\frac{\cos mx - \cos nx}{x^2} \overset{\text{H}}{=} \lim_{x\to 0}\frac{-m\sin mx + n\sin nx}{2x} \overset{\text{H}}{=} \lim_{x\to 0}\frac{-m^2\cos mx + n^2\cos nx}{2} = \tfrac{1}{2}\left(n^2-m^2\right)$$

28. This limit has the form $\infty \cdot 0$. $\lim\limits_{x\to-\infty} x^2 e^x = \lim\limits_{x\to-\infty} \frac{x^2}{e^{-x}} \overset{\text{H}}{=} \lim\limits_{x\to-\infty} \frac{2x}{-e^{-x}} \overset{\text{H}}{=} \lim\limits_{x\to-\infty} \frac{2}{e^{-x}} = \lim\limits_{x\to-\infty} 2e^x = 0$

34. This limit has the form $\infty - \infty$. $\lim\limits_{x\to0}(\csc x - \cot x) = \lim\limits_{x\to0}\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right) = \lim\limits_{x\to0} \frac{1-\cos x}{\sin x} \overset{\text{H}}{=} \lim\limits_{x\to0} \frac{\sin x}{\cos x} = 0$

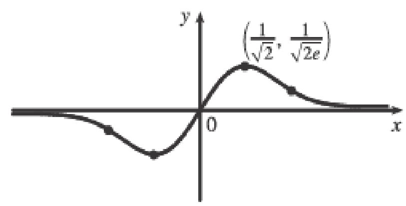
40. $y = (\tan 2x)^x \quad \Rightarrow \quad \ln y = x \cdot \ln \tan 2x$, so

$$\lim_{x\to0^+} \ln y = \lim_{x\to0^+} x \cdot \ln\tan 2x = \lim_{x\to0^+} \frac{\ln \tan 2x}{1/x} \overset{\text{H}}{=} \lim_{x\to0^+} \frac{(1/\tan 2x)(2\sec^2 2x)}{-1/x^2} = \lim_{x\to0^+} \frac{-2x^2 \cos 2x}{\sin 2x\,\cos^2 2x}$$

$$= \lim_{x\to0^+} \frac{2x}{\sin 2x} \cdot \lim_{x\to0^+} \frac{-x}{\cos 2x} = 1 \cdot 0 = 0 \quad \Rightarrow$$

$\lim\limits_{x\to0^+} (\tan 2x)^x = \lim\limits_{x\to0^+} e^{\ln y} = e^0 = 1$.

54. $\lim\limits_{x\to\pm\infty} xe^{-x^2} = \lim\limits_{x\to\pm\infty} \frac{x}{e^{x^2}} \overset{\text{H}}{=} \lim\limits_{x\to\pm\infty} \frac{1}{2xe^{x^2}} = 0$, so $y = 0$ is a HA.



$f(x) = xe^{-x^2} \quad \Rightarrow \quad f'(x) = xe^{-x^2}(-2x) + e^{-x^2} \cdot 1 = e^{-x^2}(1-2x^2) > 0 \quad \Leftrightarrow$ $x^2 < \frac{1}{2} \quad \Leftrightarrow \quad |x| < \frac{1}{\sqrt{2}}$, so f is increasing on $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and decreasing on $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{\sqrt{2}}, \infty\right)$. By the FDT, $f\left(\frac{1}{\sqrt{2}}\right) = 1/\sqrt{2e}$ is a local maximum and $f\left(-\frac{1}{\sqrt{2}}\right) = -1/\sqrt{2e}$ is a local minimum.

$f''(x) = e^{-x^2}(-4x) + (1-2x^2)e^{-x^2}(-2x) = 2xe^{-x^2}(-2-1+2x^2) = 2xe^{-x^2}(2x^2-3) > 0 \quad \Leftrightarrow \quad x > \sqrt{\frac{3}{2}}$ or $-\sqrt{\frac{3}{2}} < x < 0$, so f is CU on $\left(\sqrt{\frac{3}{2}}, \infty\right)$ and $\left(-\sqrt{\frac{3}{2}}, 0\right)$ and CD on $\left(-\infty, -\sqrt{\frac{3}{2}}\right)$ and $\left(0, \sqrt{\frac{3}{2}}\right)$. IP are $(0, 0)$ and $\left(\pm\sqrt{\frac{3}{2}}, \pm\sqrt{\frac{3}{2}}e^{-3/2}\right)$.