

2. The two numbers are $x + 100$ and x . Minimize $f(x) = (x + 100)x = x^2 + 100x$. $f'(x) = 2x + 100 = 0 \Rightarrow x = -50$.
Since $f''(x) = 2 > 0$, there is an absolute minimum at $x = -50$. The two numbers are 50 and -50 .

6. If the rectangle has dimensions x and y , then its area is $xy = 1000 \text{ m}^2$, so $y = 1000/x$. The perimeter

$P = 2x + 2y = 2x + 2000/x$. We wish to minimize the function $P(x) = 2x + 2000/x$ for $x > 0$.

$P'(x) = 2 - 2000/x^2 = (2/x^2)(x^2 - 1000)$, so the only critical number in the domain of P is $x = \sqrt{1000}$.

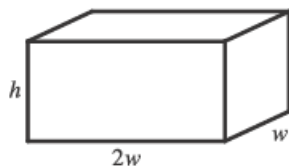
$P''(x) = 4000/x^3 > 0$, so P is concave upward throughout its domain and $P(\sqrt{1000}) = 4\sqrt{1000}$ is an absolute minimum value. The dimensions of the rectangle with minimal perimeter are $x = y = \sqrt{1000} = 10\sqrt{10} \text{ m}$. (The rectangle is a square.)

8. We need to maximize P for $I \geq 0$. $P(I) = \frac{100I}{I^2 + I + 4} \Rightarrow$

$$P'(I) = \frac{(I^2 + I + 4)(100) - 100I(2I + 1)}{(I^2 + I + 4)^2} = \frac{100(I^2 + I + 4 - 2I^2 - I)}{(I^2 + I + 4)^2} = \frac{-100(I^2 - 4)}{(I^2 + I + 4)^2} = \frac{-100(I + 2)(I - 2)}{(I^2 + I + 4)^2}$$

$P'(I) > 0$ for $0 < I < 2$ and $P'(I) < 0$ for $I > 2$. Thus, P has an absolute maximum of $P(2) = 20$ at $I = 2$.

- 14.



$$V = lwh \Rightarrow 10 = (2w)(w)h = 2w^2h, \text{ so } h = 5/w^2.$$

The cost is $10(2w^2) + 6[2(2wh) + 2(hw)] = 20w^2 + 36wh$, so

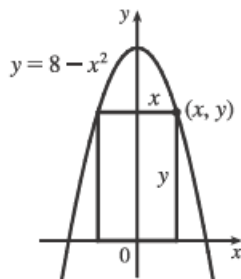
$$C(w) = 20w^2 + 36w(5/w^2) = 20w^2 + 180/w.$$

$C'(w) = 40w - 180/w^2 = 40(w^3 - \frac{9}{2})/w^2 \Rightarrow w = \sqrt[3]{\frac{9}{2}}$ is the critical number. There is an absolute minimum for C

when $w = \sqrt[3]{\frac{9}{2}}$ since $C'(w) < 0$ for $0 < w < \sqrt[3]{\frac{9}{2}}$ and $C'(w) > 0$ for $w > \sqrt[3]{\frac{9}{2}}$.

$$C\left(\sqrt[3]{\frac{9}{2}}\right) = 20\left(\sqrt[3]{\frac{9}{2}}\right)^2 + \frac{180}{\sqrt[3]{9/2}} \approx \$163.54.$$

- 18.

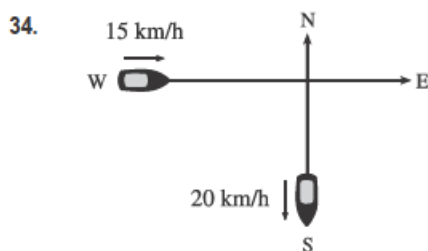


The rectangle has area $A(x) = 2xy = 2x(8 - x^2) = 16x - 2x^3$, where

$0 \leq x \leq 2\sqrt{2}$. Now $A'(x) = 16 - 6x^2 = 0 \Rightarrow x = 2\sqrt{\frac{2}{3}}$. Since

$A(0) = A(2\sqrt{2}) = 0$, there is a maximum when $x = 2\sqrt{\frac{2}{3}}$. Then $y = \frac{16}{3}$,

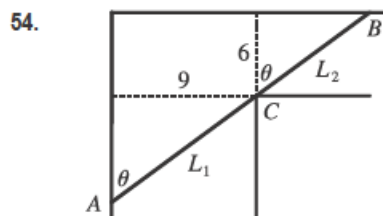
so the rectangle has dimensions $4\sqrt{\frac{2}{3}}$ and $\frac{16}{3}$.



Let t be the time, in hours, after 2:00 PM. The position of the boat heading south at time t is $(0, -20t)$. The position of the boat heading east at time t is $(-15 + 15t, 0)$. If $D(t)$ is the distance between the boats at time t , we minimize $f(t) = [D(t)]^2 = 20^2 t^2 + 15^2 (t - 1)^2$.

$$f'(t) = 800t + 450(t - 1) = 1250t - 450 = 0 \text{ when } t = \frac{450}{1250} = 0.36 \text{ h.}$$

$0.36 \text{ h} \times \frac{60 \text{ min}}{\text{h}} = 21.6 \text{ min} = 21 \text{ min } 36 \text{ s}$. Since $f''(t) > 0$, this gives a minimum, so the boats are closest together at 2:21:36 PM.



Paradoxically, we solve this maximum problem by solving a minimum problem. Let L be the length of the line ACB going from wall to wall touching the inner corner C . As $\theta \rightarrow 0$ or $\theta \rightarrow \frac{\pi}{2}$, we have $L \rightarrow \infty$ and there will be an angle that makes L a minimum. A pipe of this length will just fit around the corner.

From the diagram, $L = L_1 + L_2 = 9 \csc \theta + 6 \sec \theta \Rightarrow dL/d\theta = -9 \csc \theta \cot \theta + 6 \sec \theta \tan \theta = 0$ when

$$6 \sec \theta \tan \theta = 9 \csc \theta \cot \theta \Leftrightarrow \tan^3 \theta = \frac{9}{6} = 1.5 \Leftrightarrow \tan \theta = \sqrt[3]{1.5}. \text{ Then } \sec^2 \theta = 1 + \left(\frac{3}{2}\right)^{2/3} \text{ and}$$

$$\csc^2 \theta = 1 + \left(\frac{3}{2}\right)^{-2/3}, \text{ so the longest pipe has length } L = 9 \left[1 + \left(\frac{3}{2}\right)^{-2/3}\right]^{1/2} + 6 \left[1 + \left(\frac{3}{2}\right)^{2/3}\right]^{1/2} \approx 21.07 \text{ ft.}$$

$$\text{Or, use } \theta = \tan^{-1}(\sqrt[3]{1.5}) \approx 0.853 \Rightarrow L = 9 \csc \theta + 6 \sec \theta \approx 21.07 \text{ ft.}$$

58. Let x be the distance from the observer to the wall. Then, from the given figure,

$$\theta = \tan^{-1}\left(\frac{h+d}{x}\right) - \tan^{-1}\left(\frac{d}{x}\right), \quad x > 0 \Rightarrow$$

$$\begin{aligned} \frac{d\theta}{dx} &= \frac{1}{1 + [(h+d)/x]^2} \left[-\frac{h+d}{x^2}\right] - \frac{1}{1 + (d/x)^2} \left[-\frac{d}{x^2}\right] = -\frac{h+d}{x^2 + (h+d)^2} + \frac{d}{x^2 + d^2} \\ &= \frac{d[x^2 + (h+d)^2] - (h+d)(x^2 + d^2)}{[x^2 + (h+d)^2](x^2 + d^2)} = \frac{h^2 d + hd^2 - hx^2}{[x^2 + (h+d)^2](x^2 + d^2)} = 0 \Leftrightarrow \end{aligned}$$

$hx^2 = h^2 d + hd^2 \Leftrightarrow x^2 = hd + d^2 \Leftrightarrow x = \sqrt{d(h+d)}$. Since $d\theta/dx > 0$ for all $x < \sqrt{d(h+d)}$ and $d\theta/dx < 0$ for all $x > \sqrt{d(h+d)}$, the absolute maximum occurs when $x = \sqrt{d(h+d)}$.