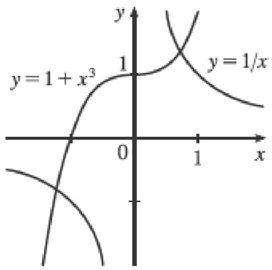
6. $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + 3 \Rightarrow f'(x) = x^2 + x$, so $x_{n+1} = x_n - \dfrac{\frac{1}{3}x_n^3 + \frac{1}{2}x_n^2 + 3}{x_n^2 + x_n}$. Now $x_1 = -3 \Rightarrow$

$$x_2 = -3 - \frac{-9 + \frac{9}{2} + 3}{9 - 3} = -3 - \left(-\tfrac{1}{4}\right) = -2.75 \Rightarrow x_3 = -2.75 - \frac{\frac{1}{3}(-2.75)^3 + \frac{1}{2}(-2.75)^2 + 3}{(-2.75)^2 + (-2.75)} \approx -2.7186.$$

12. $f(x) = x^{100} - 100 \Rightarrow f'(x) = 100x^{99}$, so $x_{n+1} = x_n - \dfrac{x_n^{100} - 100}{100x_n^{99}}$. We need to find approximations until they agree to eight decimal places. $x_1 = 1.05 \Rightarrow x_2 \approx 1.04748471$, $x_3 \approx 1.04713448$, $x_4 \approx 1.04712855 \approx x_5$.

So $\sqrt[100]{100} \approx 1.04712855$, to eight decimal places.

16.



From the graph, we see that there appear to be points of intersection near $x = -1.2$ and $x = 0.8$. Solving $\dfrac{1}{x} = 1 + x^3$ is the same as solving $f(x) = \dfrac{1}{x} - 1 - x^3 = 0$. $f(x) = \dfrac{1}{x} - 1 - x^3 \Rightarrow f'(x) = -\dfrac{1}{x^2} - 3x^2$, so

$$x_{n+1} = x_n - \frac{1/x_n - 1 - x_n^3}{-1/x_n^2 - 3x_n^2}.$$

$x_1 = -1.2$	$x_1 = 0.8$
$x_2 \approx -1.221006$	$x_2 \approx 0.724767$
$x_3 \approx -1.220744 \approx x_4$	$x_3 \approx 0.724492 \approx x_4$

To six decimal places, the roots of the equation are -1.220744 and 0.724492.

1. $f(x) = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3 \Rightarrow F(x) = \frac{1}{2}x + \dfrac{3}{4}\dfrac{x^{2+1}}{2+1} - \dfrac{4}{5}\dfrac{x^{3+1}}{3+1} + C = \frac{1}{2}x + \frac{1}{4}x^3 - \frac{1}{5}x^4 + C$

Check: $F'(x) = \frac{1}{2} + \frac{1}{4}(3x^2) - \frac{1}{5}(4x^3) + 0 = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3 = f(x)$

2. $f(x) = 8x^9 - 3x^6 + 12x^3 \Rightarrow F(x) = 8\left(\frac{1}{10}x^{10}\right) - 3\left(\frac{1}{7}x^7\right) + 12\left(\frac{1}{4}x^4\right) + C = \frac{4}{5}x^{10} - \frac{3}{7}x^7 + 3x^4 + C$

4. $f(x) = x(2-x)^2 = x\left(4 - 4x + x^2\right) = 4x - 4x^2 + x^3 \Rightarrow$

$F(x) = 4\left(\frac{1}{2}x^2\right) - 4\left(\frac{1}{3}x^3\right) + \frac{1}{4}x^4 + C = 2x^2 - \frac{4}{3}x^3 + \frac{1}{4}x^4 + C$

6. $f(x) = 2x + 3x^{1.7} \Rightarrow F(x) = x^2 + \frac{3}{2.7}x^{2.7} + C = x^2 + \frac{10}{9}x^{2.7} + C$

8. $f(x) = \sqrt[4]{x^3} + \sqrt[3]{x^4} = x^{3/4} + x^{4/3} \Rightarrow F(x) = \dfrac{x^{7/4}}{7/4} + \dfrac{x^{7/3}}{7/3} + C = \frac{4}{7}x^{7/4} + \frac{3}{7}x^{7/3} + C$

12. $f(x) = 3e^x + 7\sec^2 x \Rightarrow F(x) = 3e^x + 7\tan x + C_n$ on the interval $\left(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2}\right)$.

16. $f(x) = \dfrac{2 + x^2}{1 + x^2} = \dfrac{1 + (1 + x^2)}{1 + x^2} = \dfrac{1}{1 + x^2} + 1 \Rightarrow F(x) = \tan^{-1} x + x + C$

22. $f''(x) = 6x + \sin x \quad \Rightarrow \quad f'(x) = 6\left(\dfrac{x^2}{2}\right) - \cos x + C = 3x^2 - \cos x + C \quad \Rightarrow$

$f(x) = 3\left(\dfrac{x^3}{3}\right) - \sin x + Cx + D = x^3 - \sin x + Cx + D$

28. $f'(x) = 4/\sqrt{1-x^2} \quad \Rightarrow \quad f(x) = 4\sin^{-1} x + C$. $f\left(\frac{1}{2}\right) = 4\sin^{-1}\left(\frac{1}{2}\right) + C = 4 \cdot \frac{\pi}{6} + C$ and $f\left(\frac{1}{2}\right) = 1 \quad \Rightarrow$

$\frac{2\pi}{3} + C = 1 \quad \Rightarrow \quad C = 1 - \frac{2\pi}{3}$, so $f(x) = 4\sin^{-1} x + 1 - \frac{2\pi}{3}$.

46. For the first ball, $s_1(t) = -16t^2 + 48t + 432$ from Example 6. For the second ball, $a(t) = -32 \quad \Rightarrow \quad v(t) = -32t + C$, but $v(1) = -32(1) + C = 24 \quad \Rightarrow \quad C = 56$, so $v(t) = -32t + 56 \quad \Rightarrow \quad s(t) = -16t^2 + 56t + D$, but $s(1) = -16(1)^2 + 56(1) + D = 432 \quad \Rightarrow \quad D = 392$, and $s_2(t) = -16t^2 + 56t + 392$. The balls pass each other when $s_1(t) = s_2(t) \quad \Rightarrow \quad -16t^2 + 48t + 432 = -16t^2 + 56t + 392 \quad \Leftrightarrow \quad 8t = 40 \quad \Leftrightarrow \quad t = 5$ s.

Another solution: From Exercise 44, we have $s_1(t) = -16t^2 + 48t + 432$ and $s_2(t) = -16t^2 + 24t + 432$. We now want to solve $s_1(t) = s_2(t-1) \quad \Rightarrow \quad -16t^2 + 48t + 432 = -16(t-1)^2 + 24(t-1) + 432 \quad \Rightarrow$ $48t = 32t - 16 + 24t - 24 \quad \Rightarrow \quad 40 = 8t \quad \Rightarrow \quad t = 5$ s.