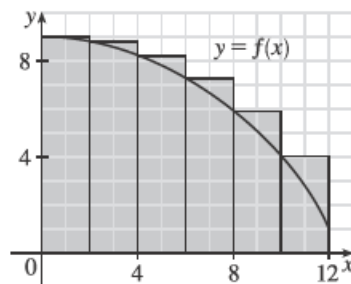
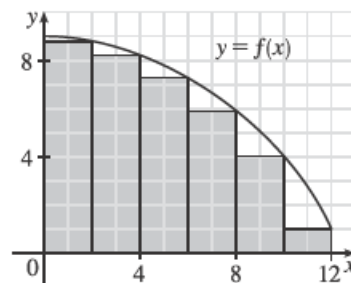


$$\begin{aligned}
 2. \text{ (a) (i) } L_6 &= \sum_{i=1}^6 f(x_{i-1}) \Delta x \quad [\Delta x = \frac{12-0}{6} = 2] \\
 &= 2[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)] \\
 &= 2[f(0) + f(2) + f(4) + f(6) + f(8) + f(10)] \\
 &\approx 2(9 + 8.8 + 8.2 + 7.3 + 5.9 + 4.1) \\
 &= 2(43.3) = 86.6
 \end{aligned}$$

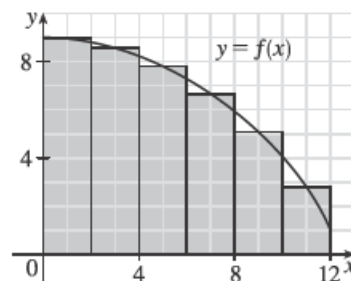


$$\begin{aligned}
 \text{(ii) } R_6 &= L_6 + 2 \cdot f(12) - 2 \cdot f(0) \\
 &\approx 86.6 + 2(1) - 2(9) = 70.6
 \end{aligned}$$

[Add area of rightmost lower rectangle
and subtract area of leftmost upper rectangle.]



$$\begin{aligned}
 \text{(iii) } M_6 &= \sum_{i=1}^6 f(x_i^*) \Delta x \\
 &= 2[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)] \\
 &\approx 2(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8) \\
 &= 2(39.7) = 79.4
 \end{aligned}$$

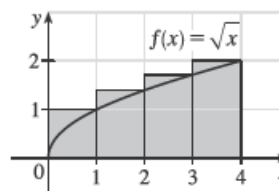


(b) Since f is decreasing, we obtain an overestimate by using left endpoints; that is, L_6 .

(c) Since f is decreasing, we obtain an underestimate by using right endpoints; that is, R_6 .

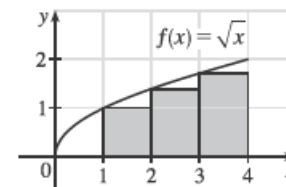
(d) M_6 gives the best estimate, since the area of each rectangle appears to be closer to the true area than the overestimates and underestimates in L_6 and R_6 .

$$\begin{aligned}
 4. \text{ (a) } R_4 &= \sum_{i=1}^4 f(x_i) \Delta x \quad \left[\Delta x = \frac{4-0}{4} = 1 \right] \\
 &= f(x_1) \cdot 1 + f(x_2) \cdot 1 + f(x_3) \cdot 1 + f(x_4) \cdot 1 \\
 &= f(1) + f(2) + f(3) + f(4) \\
 &= \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} \approx 6.1463
 \end{aligned}$$



Since f is increasing on $[0, 4]$, R_4 is an overestimate.

$$\begin{aligned}
 \text{(b) } L_4 &= \sum_{i=1}^4 f(x_{i-1}) \Delta x = f(x_0) \cdot 1 + f(x_1) \cdot 1 + f(x_2) \cdot 1 + f(x_3) \cdot 1 \\
 &= f(0) + f(1) + f(2) + f(3) \\
 &= \sqrt{0} + \sqrt{1} + \sqrt{2} + \sqrt{3} \approx 4.1463
 \end{aligned}$$



Since f is increasing on $[0, 4]$, L_4 is an underestimate.

12. (a) $d \approx L_5 = (30 \text{ ft/s})(12 \text{ s}) + 28 \cdot 12 + 25 \cdot 12 + 22 \cdot 12 + 24 \cdot 12$
 $= (30 + 28 + 25 + 22 + 24) \cdot 12 = 129 \cdot 12 = 1548 \text{ ft}$

(b) $d \approx R_5 = (28 + 25 + 22 + 24 + 27) \cdot 12 = 126 \cdot 12 = 1512 \text{ ft}$

(c) The estimates are neither lower nor upper estimates since v is neither an increasing nor a decreasing function of t .

18. $f(x) = x^2 + \sqrt{1+2x}$, $4 \leq x \leq 7$. $\Delta x = (7-4)/n = 3/n$ and $x_i = 4 + i\Delta x = 4 + 3i/n$.

$$A = \lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[(4 + 3i/n)^2 + \sqrt{1 + 2(4 + 3i/n)} \right] \cdot \frac{3}{n}.$$

20. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2}{n} \left(5 + \frac{2i}{n} \right)^{10}$ can be interpreted as the area of the region lying under the graph of $y = (5+x)^{10}$ on the interval

$[0, 2]$, since for $y = (5+x)^{10}$ on $[0, 2]$ with $\Delta x = \frac{2-0}{n} = \frac{2}{n}$, $x_i = 0 + i\Delta x = \frac{2i}{n}$, and $x_i^* = x_i$, the expression for the

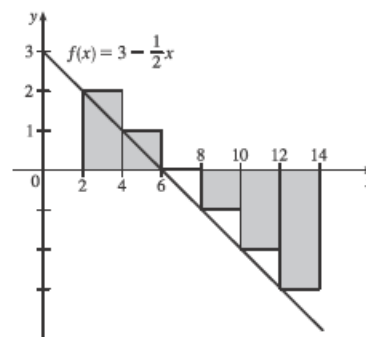
area is $A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(5 + \frac{2i}{n} \right)^{10} \frac{2}{n}$. Note that the answer is not unique. We could use $y = x^{10}$

on $[5, 7]$ or, in general, $y = ((5-n) + x)^{10}$ on $[n, n+2]$.

1. $f(x) = 3 - \frac{1}{2}x$, $2 \leq x \leq 14$. $\Delta x = \frac{b-a}{n} = \frac{14-2}{6} = 2$.

Since we are using left endpoints, $x_i^* = x_{i-1}$.

$$\begin{aligned} L_6 &= \sum_{i=1}^6 f(x_{i-1}) \Delta x \\ &= (\Delta x) [f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)] \\ &= 2[f(2) + f(4) + f(6) + f(8) + f(10) + f(12)] \\ &= 2[2 + 1 + 0 + (-1) + (-2) + (-3)] = 2(-3) = -6 \end{aligned}$$

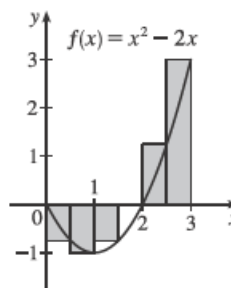


The Riemann sum represents the sum of the areas of the two rectangles above the x -axis minus the sum of the areas of the three rectangles below the x -axis; that is, the *net area* of the rectangles with respect to the x -axis.

$$2. f(x) = x^2 - 2x, \quad 0 \leq x \leq 3. \quad \Delta x = \frac{b-a}{n} = \frac{3-0}{6} = \frac{1}{2}.$$

Since we are using right endpoints, $x_i^* = x_i$.

$$\begin{aligned} R_6 &= \sum_{i=1}^6 f(x_i) \Delta x \\ &= (\Delta x) [f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)] \\ &= \frac{1}{2} [f(\frac{1}{2}) + f(1) + f(\frac{3}{2}) + f(2) + f(\frac{5}{2}) + f(3)] \\ &= \frac{1}{2} (-\frac{3}{4} - 1 - \frac{3}{4} + 0 + \frac{5}{4} + 3) = \frac{1}{2} (\frac{7}{4}) = \frac{7}{8} \end{aligned}$$



The Riemann sum represents the sum of the areas of the two rectangles above the x -axis minus the sum of the areas of the three rectangles below the x -axis; that is, the *net area* of the rectangles with respect to the x -axis.

$$5. \Delta x = (b-a)/n = (8-0)/4 = 8/4 = 2.$$

(a) Using the right endpoints to approximate $\int_0^8 f(x) dx$, we have

$$\sum_{i=1}^4 f(x_i) \Delta x = 2[f(2) + f(4) + f(6) + f(8)] \approx 2[1 + 2 + (-2) + 1] = 4.$$

(b) Using the left endpoints to approximate $\int_0^8 f(x) dx$, we have

$$\sum_{i=1}^4 f(x_{i-1}) \Delta x = 2[f(0) + f(2) + f(4) + f(6)] \approx 2[2 + 1 + 2 + (-2)] = 6.$$

(c) Using the midpoint of each subinterval to approximate $\int_0^8 f(x) dx$, we have

$$\sum_{i=1}^4 f(\bar{x}_i) \Delta x = 2[f(1) + f(3) + f(5) + f(7)] \approx 2[3 + 2 + 1 + (-1)] = 10.$$

6. (a) Using the right endpoints to approximate $\int_{-3}^3 g(x) dx$, we have

$$\sum_{i=1}^6 g(x_i) \Delta x = 1[g(-2) + g(-1) + g(0) + g(1) + g(2) + g(3)] \approx 1 - 0.5 - 1.5 - 1.5 - 0.5 + 2.5 = -0.5.$$

(b) Using the left endpoints to approximate $\int_{-3}^3 g(x) dx$, we have

$$\sum_{i=1}^6 g(x_{i-1}) \Delta x = 1[g(-3) + g(-2) + g(-1) + g(0) + g(1) + g(2)] \approx 2 + 1 - 0.5 - 1.5 - 1.5 - 0.5 = -1.$$

(c) Using the midpoint of each subinterval to approximate $\int_{-3}^3 g(x) dx$, we have

$$\begin{aligned} \sum_{i=1}^6 g(\bar{x}_i) \Delta x &= 1[g(-2.5) + g(-1.5) + g(-0.5) + g(0.5) + g(1.5) + g(2.5)] \\ &\approx 1.5 + 0 - 1 - 1.75 - 1 + 0.5 = -1.75 \end{aligned}$$

10. $\Delta x = (\pi/2 - 0)/4 = \frac{\pi}{8}$, so the endpoints are $0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8},$ and $\frac{\pi}{2}$, and the midpoints are $\frac{\pi}{16}, \frac{3\pi}{16}, \frac{5\pi}{16},$ and $\frac{7\pi}{16}$. The Midpoint Rule gives

$$\int_0^{\pi/2} \cos^4 x dx \approx \sum_{i=1}^4 f(\bar{x}_i) \Delta x = \frac{\pi}{8} [\cos^4(\frac{\pi}{16}) + \cos^4(\frac{3\pi}{16}) + \cos^4(\frac{5\pi}{16}) + \cos^4(\frac{7\pi}{16})] = \frac{\pi}{8} (\frac{3}{2}) \approx 0.5890.$$

18. On $[\pi, 2\pi]$, $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\cos x_i}{x_i} \Delta x = \int_{\pi}^{2\pi} \frac{\cos x}{x} dx$.

32. (a) $\int_0^2 g(x) dx = \frac{1}{2} \cdot 4 \cdot 2 = 4$ [area of a triangle]

(b) $\int_2^6 g(x) dx = -\frac{1}{2}\pi(2)^2 = -2\pi$ [negative of the area of a semicircle]

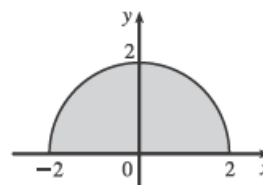
(c) $\int_6^7 g(x) dx = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$ [area of a triangle]

$$\int_0^7 g(x) dx = \int_0^2 g(x) dx + \int_2^6 g(x) dx + \int_6^7 g(x) dx = 4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi$$

34. $\int_{-2}^2 \sqrt{4-x^2} dx$ can be interpreted as the area under the graph of

$f(x) = \sqrt{4-x^2}$ between $x = -2$ and $x = 2$. This is equal to half the area of

the circle with radius 2, so $\int_{-2}^2 \sqrt{4-x^2} dx = \frac{1}{2}\pi \cdot 2^2 = 2\pi$.



47. $\int_0^3 f(x) dx$ is clearly less than -1 and has the smallest value. The slope of the tangent line of f at $x = 1$, $f'(1)$, has a value between -1 and 0 , so it has the next smallest value. The largest value is $\int_3^8 f(x) dx$, followed by $\int_4^8 f(x) dx$, which has a value about 1 unit less than $\int_3^8 f(x) dx$. Still positive, but with a smaller value than $\int_4^8 f(x) dx$, is $\int_0^8 f(x) dx$. Ordering these quantities from smallest to largest gives us

$$\int_0^3 f(x) dx < f'(1) < \int_0^8 f(x) dx < \int_4^8 f(x) dx < \int_3^8 f(x) dx \text{ or } B < E < A < D < C$$

49. $I = \int_{-4}^2 [f(x) + 2x + 5] dx = \int_{-4}^2 f(x) dx + 2 \int_{-4}^2 x dx + \int_{-4}^2 5 dx$
 $= I_1 + 2I_2 + I_3$

$$I_1 = -3 \text{ [area below } x\text{-axis]} + 3 - 3 = -3$$

$$I_2 = -\frac{1}{2}(4)(4) \text{ [area of triangle, see figure]} + \frac{1}{2}(2)(2)$$

$$= -8 + 2 = -6$$

$$I_3 = 5[2 - (-4)] = 5(6) = 30$$

Thus, $I = -3 + 2(-6) + 30 = 15$.

