1. $\int_{-2}^{3}(x^2-3)\,dx = \left[\frac{1}{3}x^3-3x\right]_{-2}^{3} = (9-9)-\left(-\frac{8}{3}+6\right)=\frac{8}{3}-\frac{18}{3}=-\frac{10}{3}$

2. $\int_1^2 x^{-2}\,dx = \left[\frac{x^{-1}}{-1}\right]_1^2 = \left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2}-(-1)=\frac{1}{2}$

7. $\int_{-1}^{0}(2x-e^x)\,dx = \left[x^2-e^x\right]_{-1}^{0} = (0-1)-\left(1-e^{-1}\right) = -2+1/e$

10. $\int_0^2 (y-1)(2y+1)\,dy = \int_0^2 (2y^2-y-1)\,dy = \left[\frac{2}{3}y^3-\frac{1}{2}y^2-y\right]_0^2 = \left(\frac{16}{3}-2-2\right)-0=\frac{4}{3}$

14. $\int_0^{\pi/4}\sec\theta\,\tan\theta\,d\theta = [\sec\theta]_0^{\pi/4} = \sec\frac{\pi}{4}-\sec 0 = \sqrt{2}-1$

16. $\displaystyle\int_1^{18}\sqrt{\frac{3}{z}}\,dz = \int_1^{18}\sqrt{3}z^{-1/2}\,dz = \sqrt{3}\left[2z^{1/2}\right]_1^{18} = 2\sqrt{3}(18^{1/2}-1^{1/2}) = 2\sqrt{3}(3\sqrt{2}-1)$

20. $\displaystyle\int_0^1 10^x\,dx = \left[\frac{10^x}{\ln 10}\right]_0^1 = \frac{10}{\ln 10}-\frac{1}{\ln 10}=\frac{9}{\ln 10}$

28. $|2x-1| = \begin{cases} 2x-1 & \text{if } 2x-1\geq 0 \\ -(2x-1) & \text{if } 2x-1<0 \end{cases} = \begin{cases} 2x-1 & \text{if } x\geq \frac{1}{2} \\ 1-2x & \text{if } x<\frac{1}{2} \end{cases}$

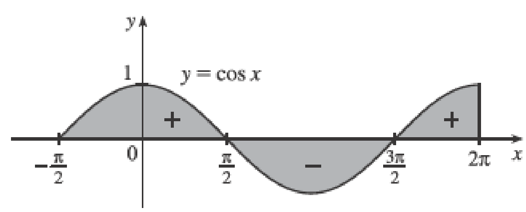
Thus, $\int_0^2 |2x-1|\,dx = \int_0^{1/2}(1-2x)\,dx + \int_{1/2}^{2}(2x-1)\,dx = \left[x-x^2\right]_0^{1/2} + \left[x^2-x\right]_{1/2}^{2}$

$= \left(\frac{1}{2}-\frac{1}{4}\right)-0+(4-2)-\left(\frac{1}{4}-\frac{1}{2}\right) = \frac{1}{4}+2-\left(-\frac{1}{4}\right)=\frac{5}{2}$

31. $f(x)=1/x^2$ is not continuous on the interval $[-1,3]$, so the Evaluation Theorem does not apply. In fact, f has an infinite discontinuity at $x=0$, so $\int_{-1}^{3}(1/x^2)\,dx$ does not exist.

38. $\int_{-\pi/2}^{2\pi}\cos x\,dx = \left[\sin x\right]_{-\pi/2}^{2\pi} = \sin 2\pi - \sin(-\pi/2)$

$= 0-(-1)=1$



44. $\displaystyle\int v(v^2+2)^2\,dv = \int v(v^4+4v^2+4)\,dv = \int (v^5+4v^3+4v)\,dv = \frac{v^6}{6}+4\frac{v^4}{4}+4\frac{v^2}{2}+C = \frac{1}{6}v^6+v^4+2v^2+C$

2. (a) $g(x) = \int_0^x f(t)\,dt$, so $g(0) = \int_0^0 f(t)\,dt = 0$.

$g(1) = \int_0^1 f(t)\,dt = \frac{1}{2}\cdot 1\cdot 1$ [area of triangle] $= \frac{1}{2}$.

$g(2) = \int_0^2 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^2 f(t)\,dt$ [below the x-axis]

$= \frac{1}{2} - \frac{1}{2}\cdot 1\cdot 1 = 0$.

$g(3) = g(2) + \int_2^3 f(t)\,dt = 0 - \frac{1}{2}\cdot 1\cdot 1 = -\frac{1}{2}$.

$g(4) = g(3) + \int_3^4 f(t)\,dt = -\frac{1}{2} + \frac{1}{2}\cdot 1\cdot 1 = 0$.

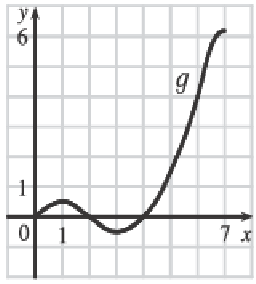
$g(5) = g(4) + \int_4^5 f(t)\,dt = 0 + 1.5 = 1.5$.

$g(6) = g(5) + \int_5^6 f(t)\,dt = 1.5 + 2.5 = 4$.

(b) $g(7) = g(6) + \int_6^7 f(t)\,dt \approx 4 + 2.2$ [estimate from the graph] $= 6.2$.

(c) The answers from part (a) and part (b) indicate that g has a minimum at $x = 3$ and a maximum at $x = 7$. This makes sense from the graph of f since we are subtracting area on $1 < x < 3$ and adding area on $3 < x < 7$.

(d)



3. (a) $g(x) = \int_0^x f(t)\,dt$.

$g(0) = \int_0^0 f(t)\,dt = 0$

$g(1) = \int_0^1 f(t)\,dt = 1\cdot 2 = 2$ [rectangle],

$g(2) = \int_0^2 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^2 f(t)\,dt = g(1) + \int_1^2 f(t)\,dt$

$= 2 + 1\cdot 2 + \frac{1}{2}\cdot 1\cdot 2 = 5$ [rectangle plus triangle],

$g(3) = \int_0^3 f(t)\,dt = g(2) + \int_2^3 f(t)\,dt = 5 + \frac{1}{2}\cdot 1\cdot 4 = 7$,

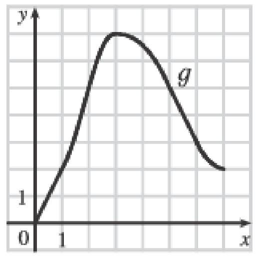
$g(6) = g(3) + \int_3^6 f(t)\,dt$ [the integral is negative since f lies under the x-axis]

$= 7 + \left[-\left(\frac{1}{2}\cdot 2\cdot 2 + 1\cdot 2\right)\right] = 7 - 4 = 3$

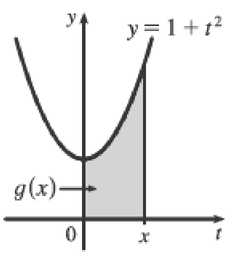
(b) g is increasing on $(0, 3)$ because as x increases from 0 to 3, we keep adding more area.

(c) g has a maximum value when we start subtracting area; that is, at $x = 3$.

(d)



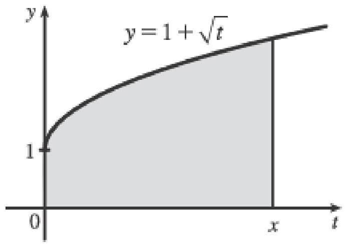
5.



(a) By FTC1, $g(x) = \int_0^x (1 + t^2)\,dt \quad \Rightarrow \quad g'(x) = f(x) = 1 + x^2$.

(b) By FTC2, $g(x) = \int_0^x (1 + t^2)\,dt = \left[t + \frac{1}{3}t^3\right]_0^x = \left(x + \frac{1}{3}x^3\right) - 0 \quad \Rightarrow \quad g'(x) = 1 + x^2$.

6.



(a) By FTC1 with $f(t) = 1 + \sqrt{t}$ and $a = 0$, $g(x) = \int_0^x \left(1 + \sqrt{t}\right) dt \Rightarrow$

$g'(x) = f(x) = 1 + \sqrt{x}$.

(b) Using FTC2, $g(x) = \int_0^x \left(1 + \sqrt{t}\right) dt = \left[t + \frac{2}{3}t^{3/2}\right]_0^x = x + \frac{2}{3}x^{3/2} \Rightarrow$

$g'(x) = 1 + x^{1/2} = 1 + \sqrt{x}$.

8. $f(t) = e^{t^2-t}$ and $g(x) = \int_3^x e^{t^2-t}\,dt$, so by FTC1, $g'(x) = f(x) = e^{x^2-x}$.

12. $G(x) = \displaystyle\int_x^1 \cos\sqrt{t}\,dt = -\int_1^x \cos\sqrt{t}\,dt \Rightarrow G'(x) = -\frac{d}{dx}\int_1^x \cos\sqrt{t}\,dt = -\cos\sqrt{x}$

14. Let $u = x^2$. Then $\dfrac{du}{dx} = 2x$. Also, $\dfrac{dh}{dx} = \dfrac{dh}{du}\dfrac{du}{dx}$, so

$$h'(x) = \frac{d}{dx}\int_0^{x^2}\sqrt{1+r^3}\,dr = \frac{d}{du}\int_0^u \sqrt{1+r^3}\,dr \cdot \frac{du}{dx} = \sqrt{1+u^3}(2x) = 2x\sqrt{1+(x^2)^3} = 2x\sqrt{1+x^6}.$$

17. $g(x) = \displaystyle\int_{2x}^{3x}\frac{u^2-1}{u^2+1}\,du = \int_{2x}^{0}\frac{u^2-1}{u^2+1}\,du + \int_0^{3x}\frac{u^2-1}{u^2+1}\,du = -\int_0^{2x}\frac{u^2-1}{u^2+1}\,du + \int_0^{3x}\frac{u^2-1}{u^2+1}\,du \Rightarrow$

$$g'(x) = -\frac{(2x)^2-1}{(2x)^2+1}\cdot\frac{d}{dx}(2x) + \frac{(3x)^2-1}{(3x)^2+1}\cdot\frac{d}{dx}(3x) = -2\cdot\frac{4x^2-1}{4x^2+1} + 3\cdot\frac{9x^2-1}{9x^2+1}$$

18. $y = \displaystyle\int_{\sin x}^{\cos x}(1+v^2)^{10}\,dv = \int_{\sin x}^{0}(1+v^2)^{10}\,dv + \int_0^{\cos x}(1+v^2)^{10}\,dv$

$$= -\int_0^{\sin x}(1+v^2)^{10}\,dv + \int_0^{\cos x}(1+v^2)^{10}\,dv \Rightarrow$$

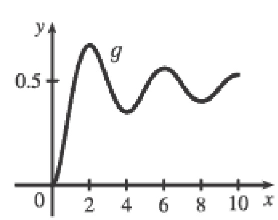
$$y' = -(1+(\sin x)^2)^{10}\cdot\frac{d}{dx}(\sin x) + (1+(\cos x)^2)^{10}\cdot\frac{d}{dx}(\cos x) = -(1+\sin^2 x)^{10}\cos x - (1+\cos^2 x)^{10}\sin x$$

20. (a) By FTC1, $g'(x) = f(x)$. So $g'(x) = f(x) = 0$ at $x = 2, 4, 6, 8$, and 10. g has local maxima at $x = 2$ and 6 (since $f = g'$ changes from positive to negative there) and local minima at $x = 4$ and 8. There is no local maximum or minimum at $x = 10$, since f is not defined for $x > 10$.

(b) We can see from the graph that $\left|\int_0^2 f\,dt\right| > \left|\int_2^4 f\,dt\right| > \left|\int_4^6 f\,dt\right| > \left|\int_6^8 f\,dt\right| > \left|\int_8^{10} f\,dt\right|$. So $g(2) = \left|\int_0^2 f\,dt\right|$, $g(6) = \int_0^6 f\,dt = g(2) - \left|\int_2^4 f\,dt\right| + \left|\int_4^6 f\,dt\right|$, and $g(10) = \int_0^{10} f\,dt = g(6) - \left|\int_6^8 f\,dt\right| + \left|\int_8^{10} f\,dt\right|$. Thus, $g(2) > g(6) > g(10)$, and so the absolute maximum of $g(x)$ occurs at $x = 2$.

(c) g is concave downward on those intervals where $g'' < 0$. But $g'(x) = f(x)$, so $g''(x) = f'(x)$, which is negative on $(1,3)$, $(5,7)$ and $(9,10)$. So g is concave downward on these intervals.

(d)



22. $g(y) = \int_3^y f(x)\,dx \quad \Rightarrow \quad g'(y) = f(y)$. Since $f(x) = \int_0^{\sin x} \sqrt{1+t^2}\,dt$, $g''(y) = f'(y) = \sqrt{1+\sin^2 y} \cdot \cos y$,

so $g''(\frac{\pi}{6}) = \sqrt{1+\sin^2(\frac{\pi}{6})} \cdot \cos\frac{\pi}{6} = \sqrt{1+(\frac{1}{2})^2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{5}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{15}}{4}$.

23. $y = \displaystyle\int_0^x \frac{t^2}{t^2+t+2}\,dt \quad \Rightarrow \quad y' = \frac{x^2}{x^2+x+2} \quad \Rightarrow$

$$y'' = \frac{(x^2+x+2)(2x) - x^2(2x+1)}{(x^2+x+2)^2} = \frac{2x^3+2x^2+4x-2x^3-x^2}{(x^2+x+2)^2} = \frac{x^2+4x}{(x^2+x+2)^2} = \frac{x(x+4)}{(x^2+x+2)^2}.$$

The curve y is concave downward when $y'' < 0$; that is, on the interval $(-4, 0)$.

30. (a) If $x < 0$, then $g(x) = \int_0^x f(t)\,dt = \int_0^x 0\,dt = 0$.

If $0 \le x \le 1$, then $g(x) = \int_0^x f(t)\,dt = \int_0^x t\,dt = \left[\frac{1}{2}t^2\right]_0^x = \frac{1}{2}x^2$.

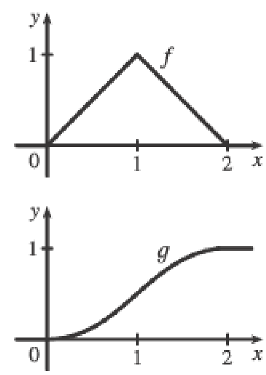
If $1 < x \le 2$, then

$$g(x) = \int_0^x f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^x f(t)\,dt = g(1) + \int_1^x (2-t)\,dt$$
$$= \tfrac{1}{2}(1)^2 + \left[2t - \tfrac{1}{2}t^2\right]_1^x = \tfrac{1}{2} + \left(2x - \tfrac{1}{2}x^2\right) - \left(2 - \tfrac{1}{2}\right) = 2x - \tfrac{1}{2}x^2 - 1.$$

If $x > 2$, then $g(x) = \int_0^x f(t)\,dt = g(2) + \int_2^x 0\,dt = 1 + 0 = 1$. So

$$g(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2}x^2 & \text{if } 0 \le x \le 1 \\ 2x - \frac{1}{2}x^2 - 1 & \text{if } 1 < x \le 2 \\ 1 & \text{if } x > 2 \end{cases}$$

(b)



(c) f is not differentiable at its corners at $x = 0$, 1, and 2. f is differentiable on $(-\infty, 0)$, $(0, 1)$, $(1, 2)$ and $(2, \infty)$.

g is differentiable on $(-\infty, \infty)$.