1. (a) If the graph of f is shifted 3 units upward, its equation becomes $y = f(x) + 3$.

   (b) If the graph of f is shifted 3 units downward, its equation becomes $y = f(x) - 3$.

   (c) If the graph of f is shifted 3 units to the right, its equation becomes $y = f(x - 3)$.

   (d) If the graph of f is shifted 3 units to the left, its equation becomes $y = f(x + 3)$.

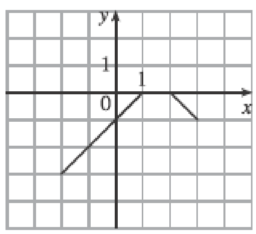
   (e) If the graph of f is reflected about the x-axis, its equation becomes $y = -f(x)$.

   (f) If the graph of f is reflected about the y-axis, its equation becomes $y = f(-x)$.

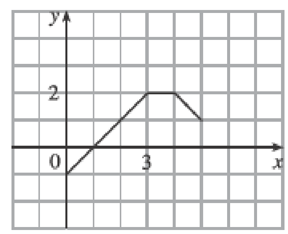
   (g) If the graph of f is stretched vertically by a factor of 3, its equation becomes $y = 3f(x)$.

   (h) If the graph of f is shrunk vertically by a factor of 3, its equation becomes $y = \frac{1}{3}f(x)$.

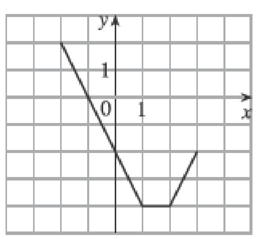
4. (a) To graph $y = f(x) - 2$, we shift the graph of f, 2 units downward.The point $(1, 2)$ on the graph of f corresponds to the point $(1, 2 - 2) = (1, 0)$.



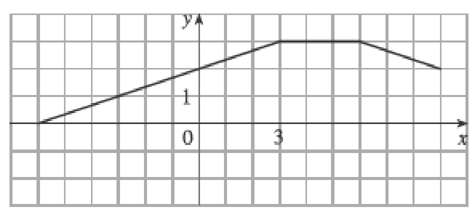
   (b) To graph $y = f(x - 2)$, we shift the graph of f, 2 units to the right.The point $(1, 2)$ on the graph of f corresponds to the point $(1 + 2, 2) = (3, 2)$.



   (c) To graph $y = -2f(x)$, we reflect the graph about the x-axis and stretch the graph vertically by a factor of 2. The point $(1, 2)$ on the graph of f corresponds to the point $(1, -2 \cdot 2) = (1, -4)$.



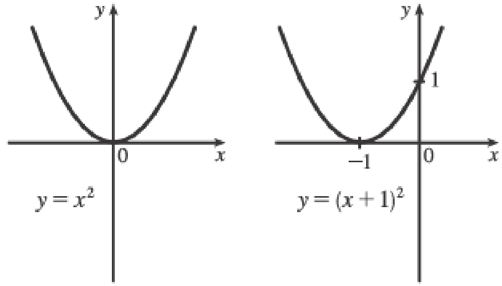
   (d) To graph $y = f(\frac{1}{3}x) + 1$, we stretch the graph horizontally by a factor of 3 and shift it 1 unit upward. The point $(1, 2)$ on the graph of f corresponds to the point $(1 \cdot 3, 2 + 1) = (3, 3)$.



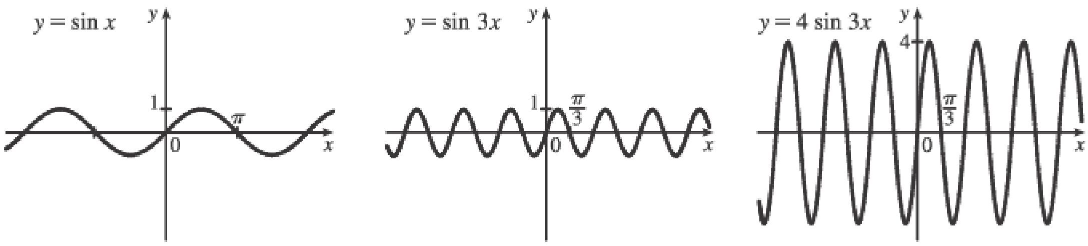
6. The graph of $y = f(x) = \sqrt{3x - x^2}$ has been shifted 2 units to the right and stretched vertically by a factor of 2. Thus, a function describing the graph is

$$y = 2f(x - 2) = 2\sqrt{3(x - 2) - (x - 2)^2} = 2\sqrt{3x - 6 - (x^2 - 4x + 4)} = 2\sqrt{-x^2 + 7x - 10}$$

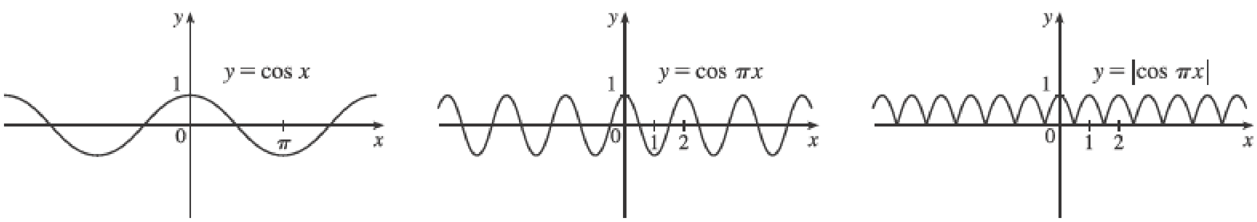
11. $y = (x+1)^2$: Start with the graph of $y = x^2$ and shift 1 unit to the left.



14. $y = 4\sin 3x$: Start with the graph of $y = \sin x$, compress horizontally by a factor of 3, and then stretch vertically by a factor of 4.



24. $y = |\cos \pi x|$: Start with the graph of $y = \cos x$, shrink it horizontally by a factor of π, and reflect all the parts of the graph below the x-axis about the x-axis.



29. $f(x) = x^3 + 2x^2$; $g(x) = 3x^2 - 1$. $D = \mathbb{R}$ for both f and g.

(a) $(f+g)(x) = (x^3 + 2x^2) + (3x^2 - 1) = x^3 + 5x^2 - 1,\ D = \mathbb{R}$.

(b) $(f-g)(x) = (x^3 + 2x^2) - (3x^2 - 1) = x^3 - x^2 + 1,\ D = \mathbb{R}$.

(c) $(fg)(x) = (x^3 + 2x^2)(3x^2 - 1) = 3x^5 + 6x^4 - x^3 - 2x^2,\ D = \mathbb{R}$.

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^3 + 2x^2}{3x^2 - 1},\ D = \left\{x \mid x \neq \pm\dfrac{1}{\sqrt{3}}\right\}$ since $3x^2 - 1 \neq 0$.

40. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(\sqrt[3]{x})) = f\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x} - 1}\right) = \tan\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x} - 1}\right)$

48. Let $h(x) = |x|$, $g(x) = 2 + x$, and $f(x) = \sqrt[8]{x}$. Then $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(|x|)) = f\,(2 + |x|) = \sqrt[8]{2 + |x|} = H(x)$.

50. (a) $f(g(1)) = f(6) = 5$

(b) $g(f(1)) = g(3) = 2$

(c) $f(f(1)) = f(3) = 4$

(d) $g(g(1)) = g(6) = 3$

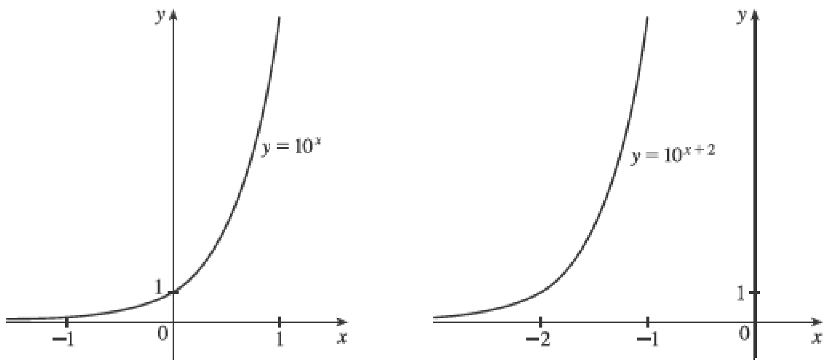
(e) $(g \circ f)(3) = g(f(3)) = g(4) = 1$

(f) $(f \circ g)(6) = f(g(6)) = f(3) = 4$

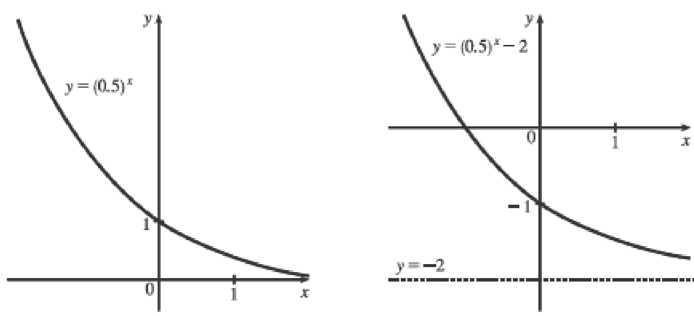
54. (a) The radius r of the balloon is increasing at a rate of $2\ \mathrm{cm/s}$, so $r(t) = (2\ \mathrm{cm/s})(t\ \mathrm{s}) = 2t$ (in cm).

(b) Using $V = \frac{4}{3}\pi r^3$, we get $(V \circ r)(t) = V(r(t)) = V(2t) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3$.
The result, $V = \frac{32}{3}\pi t^3$, gives the volume of the balloon (in cm^3) as a function of time (in s).

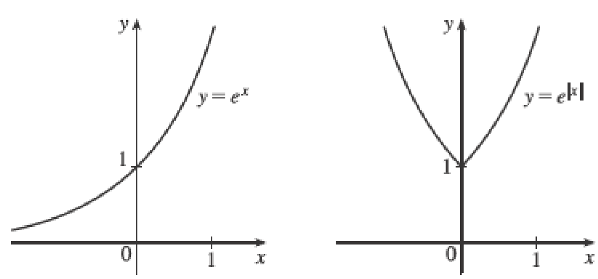
11. We start with the graph of $y = 10^x$ (Figure 3) and shift it 2 units to the left to obtain the graph of $y = 10^{x+2}$.



12. We start with the graph of $y = (0.5)^x$ (Figure 3) and shift it 2 units downward to obtain the graph of $y = (0.5)^x - 2$. The horizontal asymptote of the final graph is $y = -2$.



14. We start with the graph of $y = e^x$ (Figure 13) and reflect the portion of the graph in the first quadrant about the y-axis to obtain the graph of $y = e^{|x|}$.



18. (a) This reflection consists of first reflecting the graph about the x-axis (giving the graph with equation $y = -e^x$) and then shifting this graph $2 \cdot 4 = 8$ units upward. So the equation is $y = -e^x + 8$.

(b) This reflection consists of first reflecting the graph about the y-axis (giving the graph with equation $y = e^{-x}$) and then shifting this graph $2 \cdot 2 = 4$ units to the right. So the equation is $y = e^{-(x-4)}$.

22. Use $y = Ca^x$ with the points $(-1, 3)$ and $\left(1, \frac{4}{3}\right)$. From the point $(-1, 3)$, we have $3 = Ca^{-1}$, hence $C = 3a$. Using this and the point $\left(1, \frac{4}{3}\right)$, we get $\frac{4}{3} = Ca^1 \Rightarrow \frac{4}{3} = (3a)a \Rightarrow \frac{4}{9} = a^2 \Rightarrow a = \frac{2}{3}$ [since $a > 0$] and $C = 3(\frac{2}{3}) = 2$. The function is $f(x) = 2(\frac{2}{3})^x$.