1. (a) See Definition 1.
   (b) It must pass the Horizontal Line Test.

3. f is not one-to-one because $2 \neq 6$, but $f(2) = 2.0 = f(6)$.

6. No horizontal line intersects the graph more than once. Thus, by the Horizontal Line Test, the function is one-to-one.

12. $g(x) = \cos x$. $g(0) = 1 = g(2\pi)$, so g is not one-to-one.

16. First, we must determine x such that $f(x) = 3$. By inspection, we see that if $x = 1$, then $f(1) = 3$. Since f is 1-1 (f is an increasing function), it has an inverse, and $f^{-1}(3) = 1$. If f is a 1-1 function, then $f(f^{-1}(a)) = a$, so $f(f^{-1}(2)) = 2$.

26. $y = f(x) = \dfrac{e^x}{1+2e^x} \Rightarrow y + 2ye^x = e^x \Rightarrow y = e^x - 2ye^x \Rightarrow y = e^x(1-2y) \Rightarrow e^x = \dfrac{y}{1-2y} \Rightarrow$ $x = \ln\left(\dfrac{y}{1-2y}\right)$. Interchange x and y: $y = \ln\left(\dfrac{x}{1-2x}\right)$. So $f^{-1}(x) = \ln\left(\dfrac{x}{1-2x}\right)$. Note that the range of f and the domain of f^{-1} is $(0, \frac{1}{2})$.

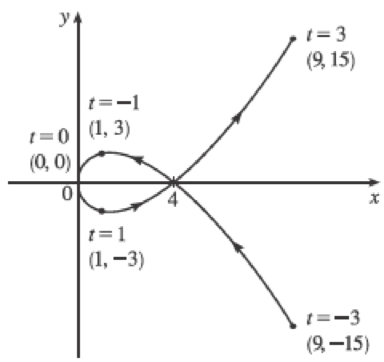
36. (a) $\ln(1/e) = \ln 1 - \ln e = 0 - 1 = -1$
    (b) $\log_{10}\sqrt{10} = \log_{10} 10^{1/2} = \frac{1}{2}$ by (7).

38. (a) $e^{-2\ln 5} = \left(e^{\ln 5}\right)^{-2} \overset{(9)}{=} 5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$
    (b) $\ln\left(\ln e^{e^{10}}\right) \overset{(9)}{=} \ln(e^{10}) \overset{(9)}{=} 10$

42. (a) $\log_{12} 10 = \dfrac{\ln 10}{\ln 12} \approx 0.926628$
    (b) $\log_2 8.4 = \dfrac{\ln 8.4}{\ln 2} \approx 3.070389$

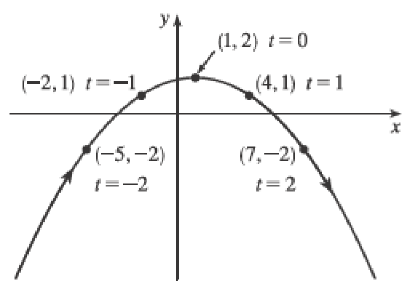
2. $x = t^2$, $y = t^3 - 4t$, $-3 \le t \le 3$

t	± 3	± 2	± 1	0
x	9	4	1	0
y	± 15	0	∓ 3	0



6. $x = 1 + 3t$, $y = 2 - t^2$
   (a)

t	-3	-2	-1	0	1	2	3
x	-8	-5	-2	1	4	7	10
y	-7	-2	1	2	1	-2	-7



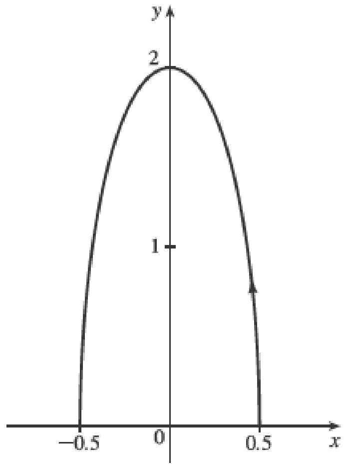
   (b) $x = 1 + 3t \Rightarrow t = \frac{1}{3}(x-1) \Rightarrow y = 2 - \left[\frac{1}{3}(x-1)\right]^2$,
   so $y = -\frac{1}{9}(x-1)^2 + 2$.

10. (a) $x = \frac{1}{2}\cos\theta$, $y = 2\sin\theta$, $0 \le \theta \le \pi$.

$(2x)^2 + \left(\frac{1}{2}y\right)^2 = \cos^2\theta + \sin^2\theta = 1 \quad \Rightarrow \quad 4x^2 + \frac{1}{4}y^2 = 1 \quad \Rightarrow$

$\dfrac{x^2}{(1/2)^2} + \dfrac{y^2}{2^2} = 1$, which is an equation of an ellipse with x-intercepts $\pm\frac{1}{2}$ and y-intercepts ± 2. For $0 \le \theta \le \pi/2$, we have $\frac{1}{2} \ge x \ge 0$ and $0 \le y \le 2$. For $\pi/2 < \theta \le \pi$, we have $0 > x \ge -\frac{1}{2}$ and $2 > y \ge 0$. So the graph is the top half of the ellipse.

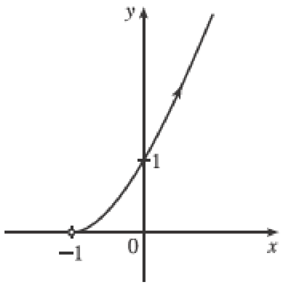
(b)



14. (a) $x = e^t - 1$, $y = e^{2t}$.

$y = (e^t)^2 = (x+1)^2$ and since $x > -1$, we have the right side of the parabola $y = (x+1)^2$.

(b)



18. $x = 2\sin t$, $y = 4 + \cos t \quad \Rightarrow \quad \sin t = \dfrac{x}{2}$, $\cos t = y - 4$. $\sin^2 t + \cos^2 t = 1 \quad \Rightarrow \quad \left(\dfrac{x}{2}\right)^2 + (y-4)^2 = 1$. The motion of the particle takes place on an ellipse centered at $(0, 4)$. As t goes from 0 to $\frac{3\pi}{2}$, the particle starts at the point $(0, 5)$ and moves clockwise to $(-2, 4)$ [three-quarters of an ellipse].

22. (a) From the first graph, we have $1 \le x \le 2$. From the second graph, we have $-1 \le y \le 1$. The only choice that satisfies either of those conditions is III.

(b) From the first graph, the values of x cycle through the values from -2 to 2 four times. From the second graph, the values of y cycle through the values from -2 to 2 six times. Choice I satisfies these conditions.

(c) From the first graph, the values of x cycle through the values from -2 to 2 three times. From the second graph, we have $0 \le y \le 2$. Choice IV satisfies these conditions.

(d) From the first graph, the values of x cycle through the values from -2 to 2 two times. From the second graph, the values of y do the same thing. Choice II satisfies these conditions.

24. For $t < -1$, x is positive and decreasing, while y is negative and increasing (these points are in Quadrant IV). When $t = -1$, $(x, y) = (0, 0)$ and, as t increases from -1 to 0, x becomes negative and y increases from 0 to 1. At $t = 0$, $(x, y) = (0, 1)$ and, as t increases from 0 to 1, y decreases from 1 to 0 and x is positive. At $t = 1$, $(x, y) = (0, 0)$ again, so the loop is completed. For $t > 1$, x and y both become large negative. This enables us to draw a rough sketch. We could achieve greater accuracy by estimating x- and y-values for selected values of t from the given graphs and plotting the corresponding points.