

2. (a)  $\lim_{x \rightarrow 2} [f(x) + g(x)] = \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} g(x) = 2 + 0 = 2$
- (b)  $\lim_{x \rightarrow 1} g(x)$  does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.
- (c)  $\lim_{x \rightarrow 0} [f(x)g(x)] = \lim_{x \rightarrow 0} f(x) \cdot \lim_{x \rightarrow 0} g(x) = 0 \cdot 1.3 = 0$
- (d) Since  $\lim_{x \rightarrow -1} g(x) = 0$  and  $g$  is in the denominator, but  $\lim_{x \rightarrow -1} f(x) = -1 \neq 0$ , the given limit does not exist.
- (e)  $\lim_{x \rightarrow 2} x^3 f(x) = \left[ \lim_{x \rightarrow 2} x^3 \right] \left[ \lim_{x \rightarrow 2} f(x) \right] = 2^3 \cdot 2 = 16$
- (f)  $\lim_{x \rightarrow 1} \sqrt{3 + f(x)} = \sqrt{3 + \lim_{x \rightarrow 1} f(x)} = \sqrt{3 + 1} = 2$

$$\begin{aligned}
 6. \lim_{u \rightarrow -2} \sqrt{u^4 + 3u + 6} &= \sqrt{\lim_{u \rightarrow -2} (u^4 + 3u + 6)} && [11] \\
 &= \sqrt{\lim_{u \rightarrow -2} u^4 + 3 \lim_{u \rightarrow -2} u + \lim_{u \rightarrow -2} 6} && [1, 2, \text{ and } 3] \\
 &= \sqrt{(-2)^4 + 3(-2) + 6} && [9, 8, \text{ and } 7] \\
 &= \sqrt{16 - 6 + 6} = \sqrt{16} = 4
 \end{aligned}$$

$$10. \lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4} = \lim_{x \rightarrow 4} \frac{x(x - 4)}{(x - 4)(x + 1)} = \lim_{x \rightarrow 4} \frac{x}{x + 1} = \frac{4}{4 + 1} = \frac{4}{5}$$

$$\begin{aligned}
 18. \lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h} &= \lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h} \cdot \frac{\sqrt{1+h} + 1}{\sqrt{1+h} + 1} = \lim_{h \rightarrow 0} \frac{(1+h) - 1}{h(\sqrt{1+h} + 1)} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{1+h} + 1)} \\
 &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{1+h} + 1} = \frac{1}{\sqrt{1+1} + 1} = \frac{1}{2}
 \end{aligned}$$

$$22. \lim_{t \rightarrow 0} \left( \frac{1}{t} - \frac{1}{t^2 + t} \right) = \lim_{t \rightarrow 0} \frac{(t^2 + t) - t}{t(t^2 + t)} = \lim_{t \rightarrow 0} \frac{t^2}{t \cdot t(t + 1)} = \lim_{t \rightarrow 0} \frac{1}{t + 1} = \frac{1}{0 + 1} = 1$$

$$34. |x + 6| = \begin{cases} x + 6 & \text{if } x + 6 \geq 0 \\ -(x + 6) & \text{if } x + 6 < 0 \end{cases} = \begin{cases} x + 6 & \text{if } x \geq -6 \\ -(x + 6) & \text{if } x < -6 \end{cases}$$

We'll look at the one-sided limits.

$$\lim_{x \rightarrow -6^+} \frac{2x + 12}{|x + 6|} = \lim_{x \rightarrow -6^+} \frac{2(x + 6)}{x + 6} = 2 \quad \text{and} \quad \lim_{x \rightarrow -6^-} \frac{2x + 12}{|x + 6|} = \lim_{x \rightarrow -6^-} \frac{2(x + 6)}{-(x + 6)} = -2$$

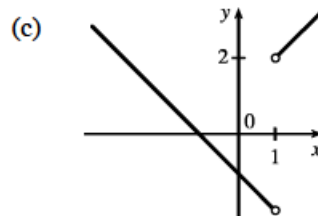
The left and right limits are different, so  $\lim_{x \rightarrow -6} \frac{2x + 12}{|x + 6|}$  does not exist.

$$36. \text{ Since } |x| = -x \text{ for } x < 0, \text{ we have } \lim_{x \rightarrow -2} \frac{2 - |x|}{2 + x} = \lim_{x \rightarrow -2} \frac{2 - (-x)}{2 + x} = \lim_{x \rightarrow -2} \frac{2 + x}{2 + x} = \lim_{x \rightarrow -2} 1 = 1.$$

$$38. (a) (i) \lim_{x \rightarrow 1^+} F(x) = \lim_{x \rightarrow 1^+} \frac{x^2 - 1}{|x - 1|} = \lim_{x \rightarrow 1^+} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1^+} (x + 1) = 2$$

$$(ii) \lim_{x \rightarrow 1^-} F(x) = \lim_{x \rightarrow 1^-} \frac{x^2 - 1}{|x - 1|} = \lim_{x \rightarrow 1^-} \frac{x^2 - 1}{-(x - 1)} = \lim_{x \rightarrow 1^-} -(x + 1) = -2$$

(b) No,  $\lim_{x \rightarrow 1} F(x)$  does not exist since  $\lim_{x \rightarrow 1^+} F(x) \neq \lim_{x \rightarrow 1^-} F(x)$ .

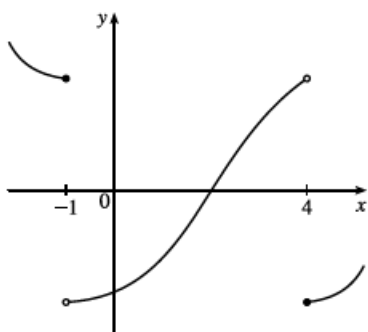


4.  $g$  is continuous on  $[-4, -2)$ ,  $(-2, 2)$ ,  $[2, 4)$ ,  $(4, 6)$ , and  $(6, 8)$ .

6. The graph of  $y = f(x)$  must have discontinuities

at  $x = -1$  and  $x = 4$ . It must show that

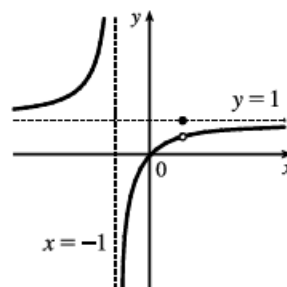
$$\lim_{x \rightarrow -1^-} f(x) = f(-1) \text{ and } \lim_{x \rightarrow 4^+} f(x) = f(4).$$



$$16. f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$$

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2 - x}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{x(x - 1)}{(x + 1)(x - 1)} = \lim_{x \rightarrow 1} \frac{x}{x + 1} = \frac{1}{2},$$

but  $f(1) = 1$ , so  $f$  is discontinuous at 1.



28. Because  $x$  is continuous on  $\mathbb{R}$ ,  $\sin x$  is continuous on  $\mathbb{R}$ , and  $x + \sin x$  is continuous on  $\mathbb{R}$ , the composite function

$$f(x) = \sin(x + \sin x) \text{ is continuous on } \mathbb{R}, \text{ so } \lim_{x \rightarrow \pi} f(x) = f(\pi) = \sin(\pi + \sin \pi) = \sin \pi = 0.$$

$$36. f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 < x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$$

$$\text{At } x = 2: \quad \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2^-} \frac{(x + 2)(x - 2)}{x - 2} = \lim_{x \rightarrow 2^-} (x + 2) = 2 + 2 = 4$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (ax^2 - bx + 3) = 4a - 2b + 3$$

We must have  $4a - b + 3 = 4$ , or  $4a - 2b = 1$  (1).

$$\text{At } x = 3: \quad \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (ax^2 - bx + 3) = 9a - 3b + 3$$

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (2x - a + b) = 6 - a + b$$

We must have  $9a - 3b + 3 = 6 - a + b$ , or  $10a - 4b = 3$  (2).

Now solve the system of equations by adding  $-2$  times equation (1) to equation (2).

$$-8a + 4b = -2$$

$$\frac{10a - 4b = 3}{2a} = 1$$

So  $a = \frac{1}{2}$ . Substituting  $\frac{1}{2}$  for  $a$  in (1) gives us  $-2b = -1$ , so  $b = \frac{1}{2}$  as well. Thus, for  $f$  to be continuous on  $(-\infty, \infty)$ ,

$$a = b = \frac{1}{2}.$$

40. Suppose that  $f(3) < 6$ . By the Intermediate Value Theorem applied to the continuous function  $f$  on the closed interval  $[2, 3]$ , the fact that  $f(2) = 8 > 6$  and  $f(3) < 6$  implies that there is a number  $c$  in  $(2, 3)$  such that  $f(c) = 6$ . This contradicts the fact that the only solutions of the equation  $f(x) = 6$  are  $x = 1$  and  $x = 4$ . Hence, our supposition that  $f(3) < 6$  was incorrect. It follows that  $f(3) \geq 6$ . But  $f(3) \neq 6$  because the only solutions of  $f(x) = 6$  are  $x = 1$  and  $x = 4$ . Therefore,  $f(3) > 6$ .

42.  $f(x) = \sqrt[3]{x} + x - 1$  is continuous on the interval  $[0, 1]$ ,  $f(0) = -1$ , and  $f(1) = 1$ . Since  $-1 < 0 < 1$ , there is a number  $c$  in  $(0, 1)$  such that  $f(c) = 0$  by the Intermediate Value Theorem. Thus, there is a root of the equation  $\sqrt[3]{x} + x - 1 = 0$ , or  $\sqrt[3]{x} = 1 - x$ , in the interval  $(0, 1)$ .