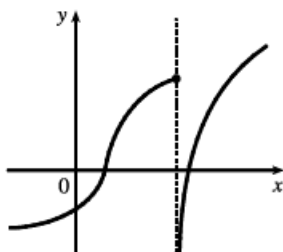
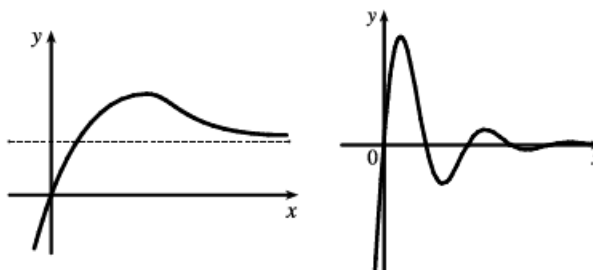


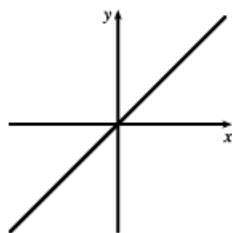
2. (a) The graph of a function can intersect a vertical asymptote in the sense that it can meet but not cross it.



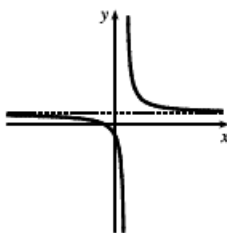
The graph of a function can intersect a horizontal asymptote. It can even intersect its horizontal asymptote an infinite number of times.



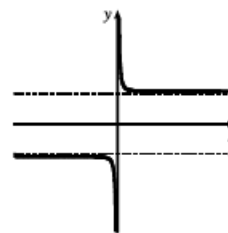
- (b) The graph of a function can have 0, 1, or 2 horizontal asymptotes. Representative examples are shown.



No horizontal asymptote



One horizontal asymptote



Two horizontal asymptotes

4. (a) $\lim_{x \rightarrow \infty} g(x) = 2$ (b) $\lim_{x \rightarrow -\infty} g(x) = -2$ (c) $\lim_{x \rightarrow 3} g(x) = \infty$
 (d) $\lim_{x \rightarrow 0} g(x) = -\infty$ (e) $\lim_{x \rightarrow -2^+} g(x) = -\infty$ (f) Vertical: $x = -2, x = 0, x = 3$; Horizontal: $y = -2, y = 2$

18. $\lim_{x \rightarrow \pi^-} \cot x = \lim_{x \rightarrow \pi^-} \frac{\cos x}{\sin x} = -\infty$ since the numerator is negative and the denominator approaches 0 through positive values as $x \rightarrow \pi^-$.

20. $\lim_{x \rightarrow 2^-} \frac{x^2 - 2x}{x^2 - 4x + 4} = \lim_{x \rightarrow 2^-} \frac{x(x-2)}{(x-2)^2} = \lim_{x \rightarrow 2^-} \frac{x}{x-2} = -\infty$ since the numerator is positive and the denominator approaches 0 through negative values as $x \rightarrow 2^-$.

$$26. \lim_{x \rightarrow \infty} \frac{x+2}{\sqrt{9x^2+1}} = \lim_{x \rightarrow \infty} \frac{(x+2)/x}{\sqrt{9x^2+1}/\sqrt{x^2}} = \lim_{x \rightarrow \infty} \frac{1+2/x}{\sqrt{9+1/x^2}} = \frac{1+0}{\sqrt{9+0}} = \frac{1}{3}$$

$$34. \text{Divide numerator and denominator by } e^{3x}: \lim_{x \rightarrow \infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}} = \lim_{x \rightarrow \infty} \frac{1 - e^{-6x}}{1 + e^{-6x}} = \frac{1-0}{1+0} = 1$$

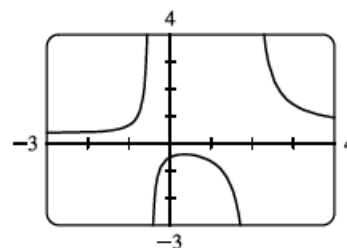
$$\begin{aligned}
 40. \lim_{x \rightarrow \infty} \frac{x^2 + 1}{2x^2 - 3x - 2} &= \lim_{x \rightarrow \infty} \frac{\frac{x^2 + 1}{x^2}}{\frac{2x^2 - 3x - 2}{x^2}} = \lim_{x \rightarrow \infty} \frac{1 + \frac{1}{x^2}}{2 - \frac{3}{x} - \frac{2}{x^2}} = \frac{\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x^2}\right)}{\lim_{x \rightarrow \infty} \left(2 - \frac{3}{x} - \frac{2}{x^2}\right)} \\
 &= \frac{\lim_{x \rightarrow \infty} 1 + \lim_{x \rightarrow \infty} \frac{1}{x^2}}{\lim_{x \rightarrow \infty} 2 - \lim_{x \rightarrow \infty} \frac{3}{x} - \lim_{x \rightarrow \infty} \frac{2}{x^2}} = \frac{1 + 0}{2 - 0 - 0} = \frac{1}{2}, \text{ so } y = \frac{1}{2} \text{ is a horizontal asymptote.}
 \end{aligned}$$

$$y = f(x) = \frac{x^2 + 1}{2x^2 - 3x - 2} = \frac{x^2 + 1}{(2x + 1)(x - 2)}, \text{ so } \lim_{x \rightarrow (-1/2)^-} f(x) = \infty$$

because as $x \rightarrow (-1/2)^-$ the numerator is positive while the denominator approaches 0 through positive values. Similarly, $\lim_{x \rightarrow (-1/2)^+} f(x) = -\infty$,

$\lim_{x \rightarrow 2^-} f(x) = -\infty$, and $\lim_{x \rightarrow 2^+} f(x) = \infty$. Thus, $x = -\frac{1}{2}$ and $x = 2$ are vertical

asymptotes. The graph confirms our work.



6. Using (2) with $f(x) = x^3 - 3x + 1$ and $P(2, 3)$,

$$\begin{aligned}
 m &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{(2+h)^3 - 3(2+h) + 1 - 3}{h} \\
 &= \lim_{h \rightarrow 0} \frac{8 + 12h + 6h^2 + h^3 - 6 - 3h - 2}{h} = \lim_{h \rightarrow 0} \frac{9h + 6h^2 + h^3}{h} = \lim_{h \rightarrow 0} \frac{h(9 + 6h + h^2)}{h} \\
 &= \lim_{h \rightarrow 0} (9 + 6h + h^2) = 9
 \end{aligned}$$

$$\text{Tangent line: } y - 3 = 9(x - 2) \Leftrightarrow y - 3 = 9x - 18 \Leftrightarrow y = 9x - 15$$

14. (a) Let
- $H(t) = 10t - 1.86t^2$
- .

$$\begin{aligned} v(1) &= \lim_{h \rightarrow 0} \frac{H(1+h) - H(1)}{h} = \lim_{h \rightarrow 0} \frac{[10(1+h) - 1.86(1+h)^2] - (10 - 1.86)}{h} \\ &= \lim_{h \rightarrow 0} \frac{10 + 10h - 1.86(1 + 2h + h^2) - 10 + 1.86}{h} \\ &= \lim_{h \rightarrow 0} \frac{10 + 10h - 1.86 - 3.72h - 1.86h^2 - 10 + 1.86}{h} \\ &= \lim_{h \rightarrow 0} \frac{6.28h - 1.86h^2}{h} = \lim_{h \rightarrow 0} (6.28 - 1.86h) = 6.28 \end{aligned}$$

The velocity of the rock after one second is 6.28 m/s.

$$\begin{aligned} \text{(b) } v(a) &= \lim_{h \rightarrow 0} \frac{H(a+h) - H(a)}{h} = \lim_{h \rightarrow 0} \frac{[10(a+h) - 1.86(a+h)^2] - (10a - 1.86a^2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{10a + 10h - 1.86(a^2 + 2ah + h^2) - 10a + 1.86a^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{10a + 10h - 1.86a^2 - 3.72ah - 1.86h^2 - 10a + 1.86a^2}{h} = \lim_{h \rightarrow 0} \frac{10h - 3.72ah - 1.86h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(10 - 3.72a - 1.86h)}{h} = \lim_{h \rightarrow 0} (10 - 3.72a - 1.86h) = 10 - 3.72a \end{aligned}$$

The velocity of the rock when $t = a$ is $(10 - 3.72a)$ m/s.

- (c) The rock will hit the surface when $H = 0 \Leftrightarrow 10t - 1.86t^2 = 0 \Leftrightarrow t(10 - 1.86t) = 0 \Leftrightarrow t = 0$ or $1.86t = 10$.

The rock hits the surface when $t = 10/1.86 \approx 5.4$ s.

- (d) The velocity of the rock when it hits the surface is $v\left(\frac{10}{1.86}\right) = 10 - 3.72\left(\frac{10}{1.86}\right) = 10 - 20 = -10$ m/s.

18. Since
- $g(5) = -3$
- , the point
- $(5, -3)$
- is on the graph of
- g
- . Since
- $g'(5) = 4$
- , the slope of the tangent line at
- $x = 5$
- is 4.

Using the point-slope form of a line gives us $y - (-3) = 4(x - 5)$, or $y = 4x - 23$.

32. Use (4) with $f(x) = \frac{4}{\sqrt{1-x}}$.

$$\begin{aligned}
 f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{\frac{4}{\sqrt{1-(a+h)}} - \frac{4}{\sqrt{1-a}}}{h} \\
 &= 4 \lim_{h \rightarrow 0} \frac{\frac{\sqrt{1-a} - \sqrt{1-a-h}}{\sqrt{1-a-h}\sqrt{1-a}}}{h} = 4 \lim_{h \rightarrow 0} \frac{\sqrt{1-a} - \sqrt{1-a-h}}{h\sqrt{1-a-h}\sqrt{1-a}} \\
 &= 4 \lim_{h \rightarrow 0} \frac{\sqrt{1-a} - \sqrt{1-a-h}}{h\sqrt{1-a-h}\sqrt{1-a}} \cdot \frac{\sqrt{1-a} + \sqrt{1-a-h}}{\sqrt{1-a} + \sqrt{1-a-h}} = 4 \lim_{h \rightarrow 0} \frac{(\sqrt{1-a})^2 - (\sqrt{1-a-h})^2}{h\sqrt{1-a-h}\sqrt{1-a}(\sqrt{1-a} + \sqrt{1-a-h})} \\
 &= 4 \lim_{h \rightarrow 0} \frac{(1-a) - (1-a-h)}{h\sqrt{1-a-h}\sqrt{1-a}(\sqrt{1-a} + \sqrt{1-a-h})} = 4 \lim_{h \rightarrow 0} \frac{-h}{h\sqrt{1-a-h}\sqrt{1-a}(\sqrt{1-a} + \sqrt{1-a-h})} \\
 &= 4 \lim_{h \rightarrow 0} \frac{-1}{\sqrt{1-a-h}\sqrt{1-a}(\sqrt{1-a} + \sqrt{1-a-h})} = 4 \cdot \frac{-1}{\sqrt{1-a}\sqrt{1-a}(\sqrt{1-a} + \sqrt{1-a})} \\
 &= \frac{-4}{(1-a)(2\sqrt{1-a})} = \frac{-2}{(1-a)^1(1-a)^{1/2}} = \frac{-2}{(1-a)^{3/2}}
 \end{aligned}$$

Note that the answers to Exercises 33–38 are not unique.

36. By Equation 5, $\lim_{x \rightarrow \pi/4} \frac{\tan x - 1}{x - \pi/4} = f'(\pi/4)$, where $f(x) = \tan x$ and $a = \pi/4$.

44. (a) (i) [2005, 2007]: $\frac{N(2007) - N(2005)}{2007 - 2005} = \frac{15,011 - 10,241}{2} = \frac{4770}{2} = 2385$ locations per year

(ii) [2005, 2006]: $\frac{N(2006) - N(2005)}{2006 - 2005} = \frac{12,440 - 10,241}{1} = 2199$ locations per year

(iii) [2004, 2005]: $\frac{N(2005) - N(2004)}{2005 - 2004} = \frac{10,241 - 8569}{1} = 1672$ locations per year

(b) Using the values from (ii) and (iii), we have $\frac{2199 + 1672}{2} = 1935.5$ locations per year.

(c) Estimating A as (2004, 8300) and B as (2006, 12,200), the slope at

2005 is $\frac{12,200 - 8300}{2006 - 2004} = \frac{3900}{2} = 1950$ locations per year.

