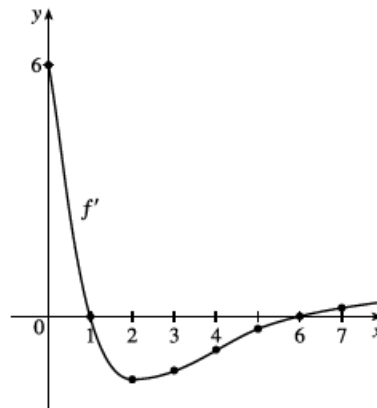


48. (a) $f'(5)$ is the rate of growth of the bacteria population when $t = 5$ hours. Its units are bacteria per hour.
- (b) With unlimited space and nutrients, f' should increase as t increases; so $f'(5) < f'(10)$. If the supply of nutrients is limited, the growth rate slows down at some point in time, and the opposite may be true.
50. (a) $f'(8)$ is the rate of change of the quantity of coffee sold with respect to the price per pound when the price is \$8 per pound. The units for $f'(8)$ are pounds/(dollars/pound).
- (b) $f'(8)$ is negative since the quantity of coffee sold will decrease as the price charged for it increases. People are generally less willing to buy a product when its price increases.
2. Your answers may vary depending on your estimates.

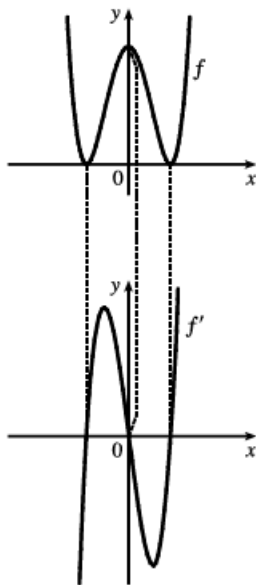
(a) *Note:* By estimating the slopes of tangent lines on the graph of f , it appears that $f'(0) \approx 6$.

- (b) $f'(1) \approx 0$
- (c) $f'(2) \approx -1.5$
- (d) $f'(3) \approx -1.3$
- (e) $f'(4) \approx -0.8$
- (f) $f'(5) \approx -0.3$
- (g) $f'(6) \approx 0$
- (h) $f'(7) \approx 0.2$



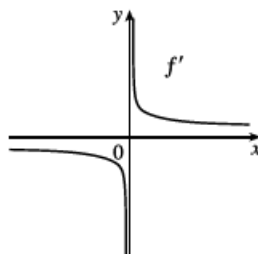
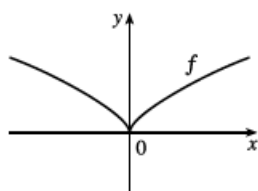
Hints for Exercises 4–11: First plot x -intercepts on the graph of f' for any horizontal tangents on the graph of f . Look for any corners on the graph of f —there will be a discontinuity on the graph of f' . On any interval where f has a tangent with positive (or negative) slope, the graph of f' will be positive (or negative). If the graph of the function is linear, the graph of f' will be a horizontal line.

4.



Hints for Exercises 4–11: First plot x -intercepts on the graph of f' for any horizontal tangents on the graph of f . Look for any corners on the graph of f —there will be a discontinuity on the graph of f' . On any interval where f has a tangent with positive (or negative) slope, the graph of f' will be positive (or negative). If the graph of the function is linear, the graph of f' will be a horizontal line.

8.



14. See Figure 1 in Section 3.3.

$$\begin{aligned} 22. f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{[1.5(x+h)^2 - (x+h) + 3.7] - (1.5x^2 - x + 3.7)}{h} \\ &= \lim_{h \rightarrow 0} \frac{1.5x^2 + 3xh + 1.5h^2 - x - h + 3.7 - 1.5x^2 + x - 3.7}{h} = \lim_{h \rightarrow 0} \frac{3xh + 1.5h^2 - h}{h} \\ &= \lim_{h \rightarrow 0} (3x + 1.5h - 1) = 3x - 1 \end{aligned}$$

Domain of f = domain of $f' = \mathbb{R}$.

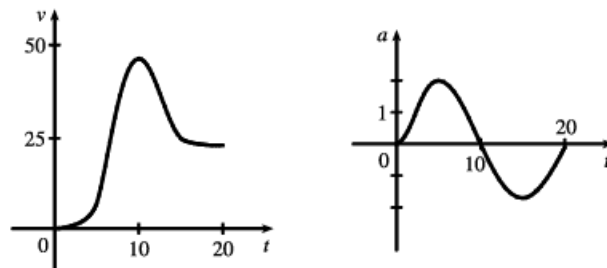
$$\begin{aligned} 24. f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h + \sqrt{x+h}) - (x + \sqrt{x})}{h} \\ &= \lim_{h \rightarrow 0} \left(\frac{h}{h} + \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right) = \lim_{h \rightarrow 0} \left[1 + \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} \right] \\ &= \lim_{h \rightarrow 0} \left(1 + \frac{1}{\sqrt{x+h} + \sqrt{x}} \right) = 1 + \frac{1}{\sqrt{x} + \sqrt{x}} = 1 + \frac{1}{2\sqrt{x}} \end{aligned}$$

Domain of $f = [0, \infty)$, domain of $f' = (0, \infty)$.

36. f is not differentiable at $x = 0$, because there is a discontinuity there, and at $x = 3$, because the graph has a vertical tangent there.

42. Where d has horizontal tangents, only c is 0, so $d' = c$. c has negative tangents for $x < 0$ and b is the only graph that is negative for $x < 0$, so $c' = b$. b has positive tangents on \mathbb{R} (except at $x = 0$), and the only graph that is positive on the same domain is a , so $b' = a$. We conclude that $d = f$, $c = f'$, $b = f''$, and $a = f'''$.

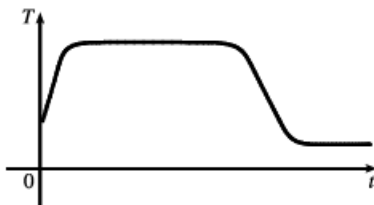
48. (a) Since we estimate the velocity to be a maximum at $t = 10$, the acceleration is 0 at $t = 10$.



- (b) Drawing a tangent line at $t = 10$ on the graph of a , a appears to decrease by 10 ft/s^2 over a period of 20 s.

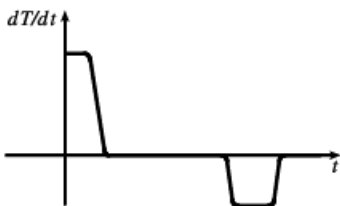
So at $t = 10$ s, the jerk is approximately $-10/20 = -0.5 \text{ (ft/s}^2\text{)/s}$ or ft/s^3 .

54. (a)



- (b) The initial temperature of the water is close to room temperature because of the water that was in the pipes. When the water from the hot water tank starts coming out, dT/dt is large and positive as T increases to the temperature of the water in the tank. In the next phase, $dT/dt = 0$ as the water comes out at a constant, high temperature. After some time, dT/dt becomes small and negative as the contents of the hot water tank are exhausted. Finally, when the hot water has run out, dT/dt is once again 0 as the water maintains its (cold) temperature.

- (c)

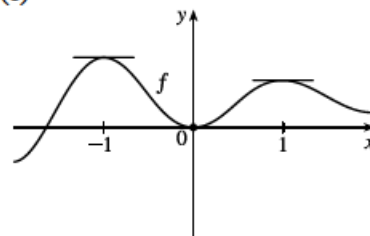


4. (a) $f'(x) > 0$ and f is increasing on $(-2, -1)$ and $(0, 1)$. $f'(x) < 0$ and f is decreasing on $(-1, 0)$ and $(1, 2)$.

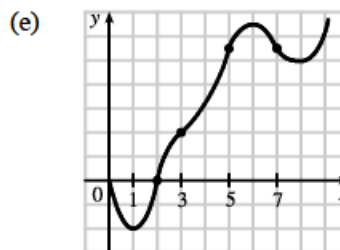
- (b) At $x = -1$ and $x = 1$, $f'(x) = 0$ and f' changes from positive to negative, so f has local maxima at $x = -1$ and $x = 1$. At $x = 0$, $f'(x) = 0$ and f' changes from negative to positive, so f has a local minimum at $x = 0$.

(The points at $x = -2$ and $x = 2$ are not part of the graph.)

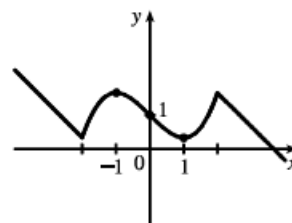
- (c)



10. (a) The rate of increase of the population is initially very small, then gets larger until it reaches a maximum at about $t = 8$ hours, and decreases toward 0 as the population begins to level off.
- (b) The rate of increase has its maximum value at $t = 8$ hours.
- (c) The population function is concave upward on $(0, 8)$ and concave downward on $(8, 18)$.
- (d) At $t = 8$, the population is about 350, so the inflection point is about $(8, 350)$.
12. (a) If the position function is increasing, then the particle is moving toward the right. This occurs on t -intervals $(0, 2)$ and $(4, 6)$. If the function is decreasing, then the particle is moving toward the left—that is, on $(2, 4)$.
- (b) The acceleration is the second derivative and is positive where the curve is concave upward. This occurs on $(3, 6)$. The acceleration is negative where the curve is concave downward—that is, on $(0, 3)$.
16. (a) f is increasing where f' is positive, on $(1, 6)$ and $(8, \infty)$, and decreasing where f' is negative, on $(0, 1)$ and $(6, 8)$.
- (b) f has a local maximum where f' changes from positive to negative, at $x = 6$, and local minima where f' changes from negative to positive, at $x = 1$ and at $x = 8$.
- (c) f is concave upward where f' is increasing, that is, on $(0, 2)$, $(3, 5)$, and $(7, \infty)$, and concave downward where f' is decreasing, that is, on $(2, 3)$ and $(5, 7)$.
- (d) There are points of inflection where f changes its direction of concavity, at $x = 2$, $x = 3$, $x = 5$ and $x = 7$.



22. $f'(1) = f'(-1) = 0 \Rightarrow$ horizontal tangents at $x = \pm 1$.
- $f'(x) < 0$ if $|x| < 1 \Rightarrow f$ is decreasing on $(-1, 1)$.
- $f'(x) > 0$ if $1 < |x| < 2 \Rightarrow f$ is increasing on $(-2, -1)$ and $(1, 2)$.
- $f'(x) = -1$ if $|x| > 2 \Rightarrow$ the graph of f has constant slope -1 on $(-\infty, -2)$ and $(2, \infty)$.
- $f''(x) < 0$ if $-2 < x < 0 \Rightarrow f$ is concave downward on $(-2, 0)$. The point $(0, 1)$ is an inflection point.



30. We know right away that c cannot be f 's antiderivative, since the slope of c is not zero at the x -value where $f = 0$. Now f is positive when a is increasing and negative when a is decreasing, so a is the antiderivative of f .