Math 210 Midterm 1 Answers

Problem 1 (4 pts) *Find the domain of the function*

$$\frac{x^2 + \sqrt{x-4}}{x-9}$$

Since we cannot take the square root of a negative number, we must have $x - 4 \geq 0$. Since we cannot divide by zero, we must also have $x \neq 9$.

So the domain is the set of all numbers x with $x \geq 4$ and $x \neq 9$.

Another way of writing this would be $[4, 9) \cup (9, \infty)$.

Problem 2 (4 pts) *Find the range of the function*

$$100^x + 8$$

Since 100^x has range $(0, \infty)$, we know that the range of this function is $(8, \infty)$.

Problem 3 (2 pts) *For*

$$f(x) = 10x + 5$$

find $f(2)$.

We plug in: $f(2) = 10 \cdot 2 + 5 = 20 + 5 = 25$.

Problem 4 (6 pts) *Each of the following functions is constructed by taking two "simple" functions f and g, and then putting them together with an operation such as $f+g$, $f-g$, $f \cdot g$, $f \div g$, or $f \circ g$. Give the operation, and f, and g.*

function	*operation*	f	g
$e^x + x^2$	$+$	e^x	x^2
$x \ln(x)$	$\cdot$	x	$\ln(x)$
$\sqrt{9x+1}$	$\circ$	$\sqrt{x}$	$9x+1$

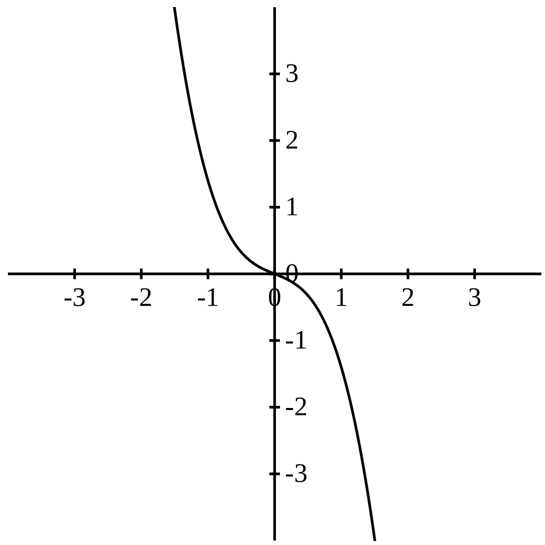
Problem 5 (4 pts) *The following function can be written as $f \circ g \circ h$, where f, g, and h are among the "simple" functions described in section 1.2. Identify f, g, and h:*

$$\sin\left(2^{(x^2)}\right)$$

$$\begin{aligned} f(x) &= \sin(x) \\ g(x) &= 2^x \\ h(x) &= x^2 \end{aligned}$$

Problem 6 (2 pts) *On the axes below, sketch a graph of an odd function that is decreasing.*

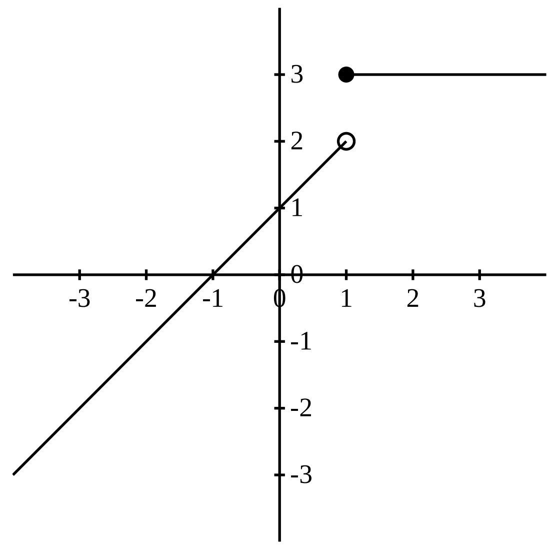
There are many possible answers here. This is one.



Problem 7 (3 pts) *On the axes below, sketch a graph of the function*

$$f(x) = \begin{cases} x+1, & x < 1 \\ 3, & x \geq 1 \end{cases}$$

To the left of $x = 1$, we have the graph of $y = x + 1$, a straight line of slope 1 and y-intercept 1. To the right of $x = 1$ we have a constant function which is a line at $y = 3$. The closed circle indicates that $f(1) = 3$.



Problem 8 (5 pts) *Simplify the following expression as far as possible:*

$$\left(a^b + b^a b^{-a}\right)\left(c^b\right)^{2/b}$$

Using the laws of exponenets, we get:

$$\begin{aligned}\left(a^b + b^a b^{-a}\right)\left(c^b\right)^{2/b} &= \left(a^b + b^{a-a}\right)\left(c^{b\cdot\frac{2}{b}}\right)\\ &= \left(a^b + b^0\right)\left(c^{1\cdot\frac{2}{1}}\right)\\ &= \left(a^b + 1\right)c^2\end{aligned}$$

Problem 9 (5 pts) *Simplify the following expression as far as possible:*

$$\ln\left(\frac{5\cdot x^x}{e^x}\right)$$

Using laws of logarithms, we get:

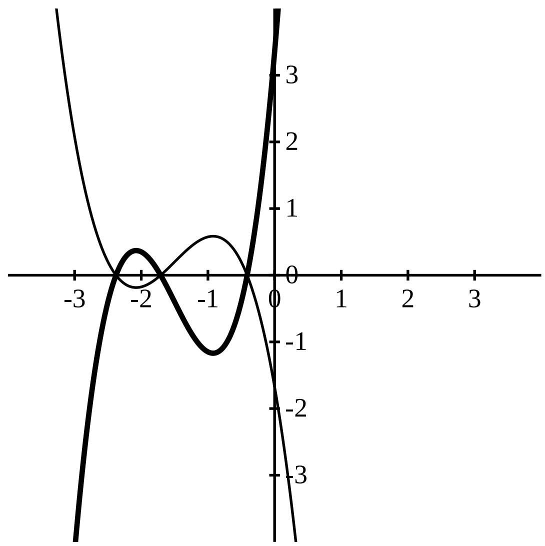
$$\begin{aligned}\ln\left(\frac{5\cdot x^x}{e^x}\right) &= \ln\left(5\cdot x^x\right) - \ln\left(e^x\right)\\ &= \ln(5) + \ln(x^x) - \ln(e^x)\\ &= \ln(5) + x\ln(x) - x\end{aligned}$$

Problem 10 (3 pts) *Suppose $f(x)$ is a function you are interested in, and suppose you wanted to move the graph of the function $f(x)$ up by 2 units and to the right by 7 units. What function could you graph to accomplish this?*

$$f(x-7)+2$$

Problem 11 (2 pts) *Below is a graph of $g(x)$. Sketch a graph of $-2\,g(x)$ on the same axes.*

This involves reflecting across the x axis and stretching by a factor of 2:



Problem 12 (5 pts) *Circle either True or False for each.*

FALSE *The function $y = x^3$ is an exponential function.* **No, it is a power function, of the form x^n.**

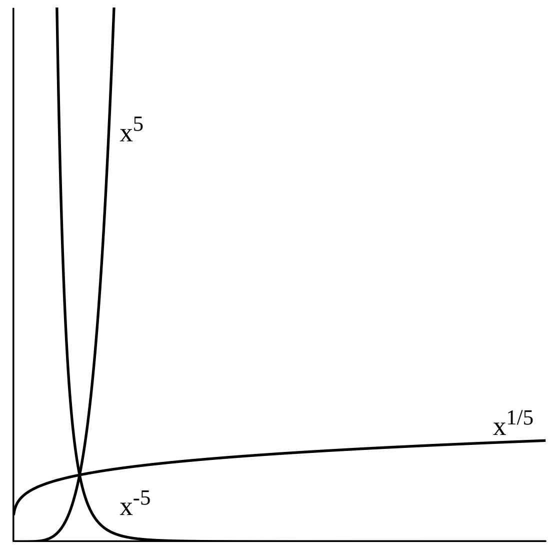
TRUE $y = e^x$ *is an example of a one-to-one function.* **Yes, it passes the horizontal line test.**

FALSE *Every function has an inverse.* **No, it must be one-to-one.**

FALSE $\ln(a+b) = \ln(a) + \ln(b)$. **This is not a law of logarithms.**

TRUE $\ln(x)$ *means* $\log_e(x)$. **Yes, that is what is meant by** $\ln$**.**

Problem 13 (5 pts) *In the graph below, identify which is x^5, $x^{1/5}$, and x^{-5}.*



Problem 14 (5 pts) *Sketch the parametric curve for $0 \leq t \leq 1$. Be sure to include numbers on the axes so we know where this curve is.*

$$\begin{aligned} x(t) &= 2t \\ y(t) &= t+1 \end{aligned}$$

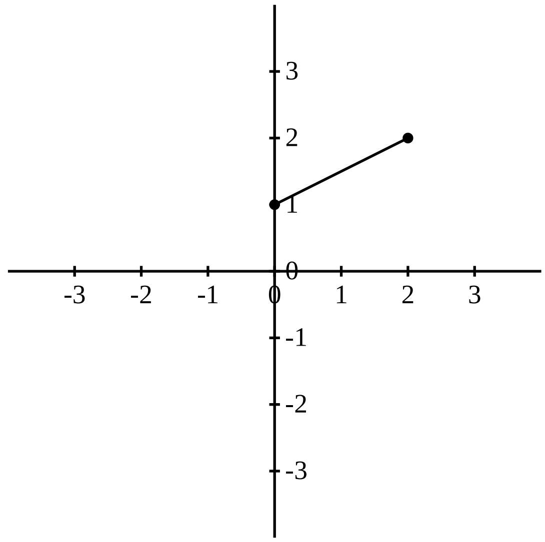
We can solve for t:

$$x/2 = t$$

and plug it into the y equation:

$$y = \frac{1}{2}t + 1$$

so the result is a line. We take $t = 0$ and get $(0,1)$. For $t = 1$ we get $(2,2)$. So for $0 \leq t \leq 1$ we trace out the line from $(0,1)$ to $(2,2)$. Note that this matches our observation that this is a line of slope $1/2$ and y-intercept 1.



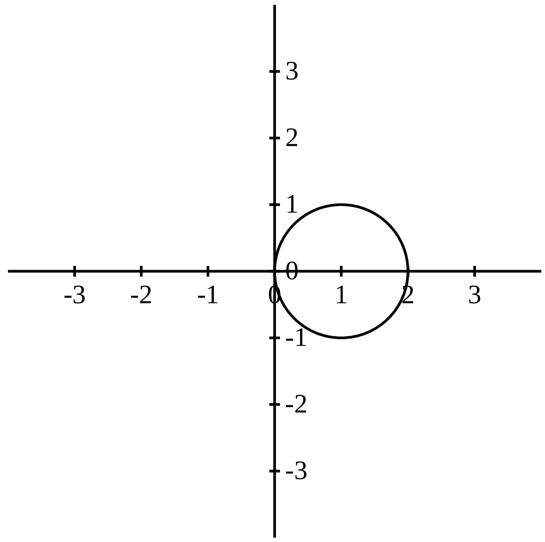
Problem 15 (5 pts) *Sketch the parametric curve for $0 \le t \le 2\pi$. Be sure to include numbers on the axes so we know where this curve is.*

$$\begin{aligned} x(t) &= 1 + \cos(2t) \\ y(t) &= \sin(2t) \end{aligned}$$

First, we recognize that

$$\begin{aligned} x(t) &= \cos(t) \\ y(t) &= \sin(t) \end{aligned}$$

is a unit circle centered at the origin. Replacing t with $2t$ means we are tracing this faster but it is the same circle. Adding 1 to x shifts everything 1 unit to the right.



Problem 16 (2 pts) *Express the following using only* ln, *not logarithms of other bases.*

$$\log_x(\sin x)$$

Using the change of base formula, we get

$$\frac{\ln(\sin x)}{\ln x}$$

Problem 17 (3 pts) *If* $\ln a = b$ *and* $\ln c = d$, *simplify the following as much as possible:*

$$\ln(ac^2) + \ln(a + c)$$

Using laws of logarithms,

$$\begin{aligned} \ln(ac^2) + \ln(a + c) &= \ln(a) + \ln(c^2) + \ln(a + c) \\ &= \ln(a) + 2\ln(c) + \ln(a + c) \\ &= b + 2d + \ln(a + c) \end{aligned}$$

Problem 18 (5 pts) *Below is a table for* $f(x)$. *Find* $f^{-1}(5)$.

x	$f(x)$
0	20
1	18
2	15
3	10
4	5
5	2
6	0

We are asking f of what is 5. By this table, f of 4 is 5. Therefore $f^{-1}(5) = 4$.

Problem 19 (5 pts) *Find the inverse of*

$$f(x) = 10 \cdot e^x.$$

We write

$$y = 10 \cdot e^x$$

and switch x and y:

$$x = 10 \cdot e^y$$

Now solve for y:

$$\begin{aligned} x &= 10 \cdot e^y \\ \frac{x}{10} &= e^y \\ \ln\left(\frac{x}{10}\right) &= y \end{aligned}$$

So

$$f^{-1}(x) = \ln\left(\frac{x}{10}\right)$$

Problem 20 (5 pts) *In statistics, the error function, abbreviated erf(x), is a one-to-one function whose domain is all real numbers, and whose range is the interval $(0,1)$. Let erf$^{-1}(x)$ be its inverse. Find the domain and range of erf^{-1}.*

The domain and range switch for inverses, so:

domain:	$(0,1)$
range:	all real numbers

Problem 21 (5 pts) *At the surface of the ocean, the water pressure is 15 psi. Below the surface, water pressure increases by 4 psi for every 10 feet of descent. Write a formula that describes the water pressure P in psi as a function of d, the depth in feet.*

This is a linear function, with slope 4/10 and y-intercept 15:

$$P(d) = \frac{4}{10}d + 15$$

Problem 22 (5 pts) *For which values of b is b^x an increasing function? (Give your answer as an interval or as a condition on b).*

For $b > 1$.

Problem 23 (5 pts) *Suppose $f(t)$ is a power function and we know $f(2) = 5$. Find a formula for $f(t)$.*

A power function is of the form

$$f(t) = t^n$$

In this case, we know $f(2) = 5$, so plugging in $t = 5$ into this equation we get

$$5 = 2^n$$

In other words, $n = \log_2(5)$. We conclude that

$$f(t) = t^{\log_2(5)}$$

Problem 24 (5 pts) *Let*

$$f(x) = x^7 + x^3 + x + 5$$

Find

$$f \circ f^{-1} \circ f \circ f^{-1} \circ f \circ f^{-1} \circ f \circ f^{-1} \circ f \circ f^{-1}(2)$$

Since the f and f^{-1} undo each other, each f cancels the next f^{-1}, so that in the end, we just get the answer 2.