Math 210 Problem set 1 Answers

Do not refer to this until you have made a good effort at working out all the problems.

Problem 1

$$(x-3)^2$$

Answer 1

$$x^2-6x+9$$

Explanation 1 *We can write the given expression as*

$$(x-3)(x-3)$$

and then distribute (sometimes this is known by the acronym FOIL, which stands for First, Outer, Inner, Last):

$$x^2+-3x+-3x+(-3)^2$$

which simplifies to

$$x^2-6x+9.$$

If you wrote

$$x^2-3^2$$

or something along those lines, remember that exponentiation does not distribute over addition, like multiplication does.

Problem 2

$$\left(-c\sqrt{98}\right)^2$$

Answer 2

$$98c^2$$

Explanation 2 *We can distribute exponents over multiplication, though. So we get*

$$(-c)^2\cdot(\sqrt{98})^2$$

and then note that minus signs go away when squared:

$$c^2\cdot(\sqrt{98})^2$$

Furthermore, square and square roots are opposite operations, so they cancel:

$$c^2\cdot 98$$

Problem 3

$$\frac{c^9}{c^{14}}$$

Answer 3

$$c^{-5}$$

Also acceptable:

$$\frac{1}{c^5}$$

Explanation 3 *We use the second law of exponents. In case you forgot, here are the three laws of exponents:*

$$\begin{aligned} x^a x^b &= x^{a+b} \\ \frac{x^a}{x^b} &= x^{a-b} \\ (x^a)^b &= x^{ab} \end{aligned}$$

One way to think of this is that exponentiation turns addition into multiplication, subtraction into division, and multiplication into exponentiation. Anyway, in our case we use the second one to get

$$c^{9-14}$$

or simplifying further,

$$c^{-5}$$

Problem 4

$$\log_{10}(1000)$$

Answer 4

$$3.$$

Explanation 4 *This tests your knowledge of logarithms. Recall that*

$$\log_b x$$

means: the number n so that $b^n = x$. In other words, what power do I have to raise b to to get x? In this case, we are asking what power we raise 10 to to get 1000. As (I hope) you recall, 10 to the third power is 1000. Therefore, the answer is 3.

Problem 5

$$\sqrt{x^2+9}$$

Answer 5

$$\sqrt{x^2+9}$$

Explanation 5 *There is nothing you can do to simplify this further. If you wrote something like $x+3$, then you were probably thinking (or hoping) that square roots distribute over addition. They do not. Leave this as is.*

Problem 6

$$\log_2(a+b)$$

Answer 6

$$\log_2(a+b)$$

Explanation 6 *Same here. There is nothing you can do. Now you should know laws of logarithms (described below), but no law of logarithm will allow you to do anything here.*

Problem 7 *If $\log_a(b) = c$ and $\log_a(c) = d$, find*

$$\log_a(bc^2).$$

Answer 7

$$c + 2d$$

Explanation 7 *Recall that each of the laws of exponents comes with a corresponding law of logarithm:*

$$\begin{aligned}\log_b(xy) &= \log_b x + \log_b y \\ \log_b(x/y) &= \log_b x - \log_b y \\ \log_b(x^n) &= n \log_b x\end{aligned}$$

Using the first law of logarithm, we can simplify the above expression as

$$\log_a b + \log_a(c^2).$$

Then we use the third law of logarithms to get

$$\log_a b + 2 \log_a c$$

Now we substitute knowing that $\log_a(b) = c$ *and* $\log_a(c) = d$*:*

$$c + 2d$$

Problem 8 *Solve*

$$(x - 3)(2x + 50) = 0.$$

Answer 8

$$x = 3 \text{ or } x = -25$$

Explanation 8 *For this equation to be true, we must have* $(x - 3) = 0$ *or* $(2x + 50) = 0$*. That is because whenever* $ab = 0$*, it must be that either* $a = 0$ *or* $b = 0$*. So we get*

$$x = 3 \text{ or } x = -25$$

In general, the reason to have factored expressions is to help us solve equations when the other side is equal to 0.

Problem 9 *Solve*

$$x(x + 2) = 3.$$

Answer 9

$$x = 1 \text{ or } x = -3.$$

Explanation 9 *Note that you cannot just say* $x = 3$ *or* $x + 2 = 3$*. That is, just because* $ab = 3$ *does not mean that* $a = 3$ *or* $b = 3$*. For instance* a *could be 38 and* b *could be 3/38. In fact, there are infinitely many cases. So this is not the way to go. In general, it makes no sense to factor unless the other side of the equation is equal to zero.*

How do we solve this, then? Well, multiply it out:

$$x^2 + 2x = 3$$

Then move the 3 to the left side:

$$x^2 + 2x - 3 = 0$$

We can factor this:

$$(x+3)(x-1)=0$$

so that $x=-3$ *or* $x=1$.

If we didn't recognize the factors we can always use the quadratic formula, which says that if we have an equation of the type

$$ax^2+bx+c=0$$

then the solutions are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Shockingly, some algebra teachers do not require students to memorize this formula. Yet, this formula can always be used to solve any quadratic polynomial equation—very useful, in a bind! If you don't know this formula, memorize it. In this case we would get

$$x=\frac{-2\pm\sqrt{2^2-4(2)(3)}}{2}=\frac{-2\pm\sqrt{16}}{2}=\frac{-2\pm 4}{2}.$$

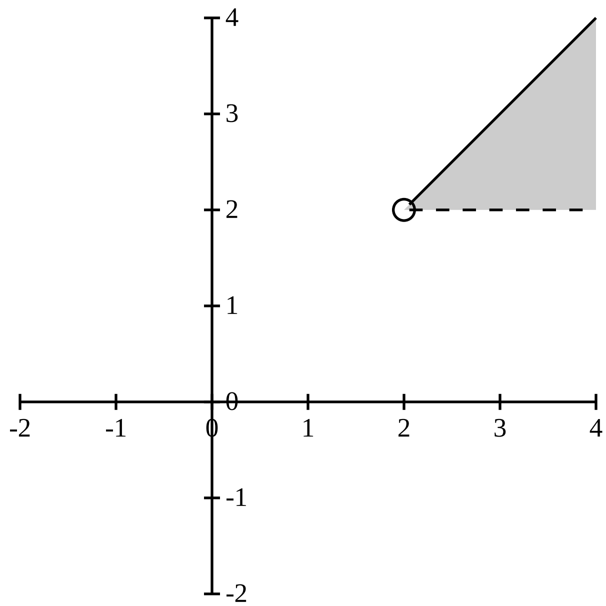
If we take the $+$ *choice we get 1, and if we choose the* $-$ *choice we get* -3.

So $x=1$ *or* $x=-3$.

Problem 10 *Sketch the region in the plane described by*

$$2<y\leq x.$$

Answer 10



Explanation 10 *This is actually two inequalities:*

$$2<y$$

and

$$y\leq x$$

that must both be satisfied. The first says $y > 2$, which means the points in the region must have height greater than 2. That defines a horizontal line that crosses the y axis at 2. The second says $y \leq x$. If we turn this into the corresponding equation $y = x$, that defines the 45 degree line through the origin as shown above. Turning it into $y \leq x$ requires the region to be the one below this line. Putting this together we get that the region is the one shown.

Since $y \leq x$ it is possible for $y = x$, so this line is included, but since $y > 2$ does not include $y = 2$, we do not include that line (hence the dashed line there). The corner $(2, 2)$ is not included since again, $y > 2$. So there is an open circle drawn to indicate that.

Problem 11 *Find all values of x for which*

$$\sin x = 0.$$

Note that there are many, many such values. Choose radians or degrees, but you must specify your answer in such a way that indicates which unit you used.

Answer 11

$$x = n\pi, \ n \text{ is an integer.}$$

This is in radians.

Explanation 11 *This is something you just have to remember. Another way of writing this is*

$$x = 0 + 2n\pi, \ \text{or } x = \pi + 2n\pi, \ n \text{ is an integer.}$$

This is more complicated but is in the form that is more typical of problems like these. In general, you should know sines and cosines of $0, 30, 45, 60,$ and 90 degrees and their corresponding angles in other quadrants; you should know these in radians as well. Also, you should recall that trigonometric functions are periodic, so the same values occur over and over again.

Problem 12 *How many radians is 110 degrees? (An expression involving π is fine).*

Answer 12

$$\frac{11\pi}{18}$$

Explanation 12 *The conversion is π radians equals 180 degrees. So we multiply by $\frac{\pi}{180°}$. This gives us*

$$\frac{110\pi}{180}$$

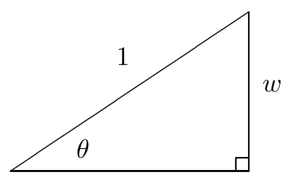
which simplifies to the above.

Problem 13 *Suppose θ is an angle so that $\sin\theta = w$. Find $\cos\theta$, $\tan\theta$, $\sec\theta$, and $\csc\theta$. Your answer will involve w. Note that for some of these, there may be more than one answer.*

Answer 13

$$\begin{aligned} \cos\theta &= \pm\sqrt{1 - w^2} \\ \tan\theta &= \pm\frac{w}{\sqrt{1 - w^2}} \\ \sec\theta &= \pm\frac{1}{\sqrt{1 - w^2}} \\ \csc\theta &= \frac{1}{w} \end{aligned}$$

Explanation 13 *One way to recreate these expressions would be to remember various trigonometric identities; you should review your trigonometric identities, but in this case, a pretty cool trick is to just draw a right triangle, and remember what* sin *means:*



We recall that $\sin\theta = w$ *means that* w *is the opposite length over the hypotenuse. That means if I draw a right triangle with* 1 *as the hypotenuse, then the opposite length to* θ *will be* w*.*

We now use the Pythagorean theorem to see that the adjacent side is $\sqrt{1-w^2}$*. That makes*

$$\begin{aligned}
\cos\theta &= \sqrt{1-w^2} \\
\tan\theta &= \frac{w}{\sqrt{1-w^2}} \\
\sec\theta &= \frac{1}{\sqrt{1-w^2}} \\
\csc\theta &= \frac{1}{w}
\end{aligned}$$

But the one problem is that I assumed θ *was in the first quadrant (at least, otherwise I can't draw the triangle). So we think about what happens when* θ *is in other quadrants. The fact is that in the other quadrants, it is possible to have the same value of* sin *but different signs of cosine, tangent, cotangent, and secant; so we need to recognize this by putting the* $\pm$ *sign on those. But cosecant needs no* $\pm$ *sign since* $\csc\theta$ *is simply* $1/\sin\theta$*.*

Problem 14 *Solve the following equation for* y*:*

$$3x + 8x^3 + \frac{8}{3x}y = 7$$

Answer 14

$$y = \frac{(7-3x-8x^3)3x}{8}$$

Explanation 14

$$\begin{aligned}
3x + 8x^3 + \frac{8}{3x}y &= 7 \\
\frac{8}{3x}y &= 7 - 3x - 8x^3 \\
y &= (7-3x-8x^3)\frac{3x}{8}
\end{aligned}$$

Problem 15 *Find the slope of the line that passes through the points* $(2,3)$ *and* $(5,12)$*.*

Answer 15

$$3.$$

Explanation 15 *The slope is*

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

which in this case is

$$\frac{12-3}{5-2} = \frac{9}{3} = 3.$$

Problem 16 *Find the equation of a line that passes through the point* $(3, 5)$ *and has slope* -8.

Answer 16

$$y = 5 - 8(x - 3)$$

Explanation 16 *This uses the point-slope formula:*

$$y = y_0 + m(x - x_0)$$

Some students think that the only way to write a formula for a line is $y = mx + b$ *and endeavor to put all their lines in this form. This is not necessary and in some cases, is counter-productive. It is perfectly acceptable to leave the equation in the form given above. In case you did that, though, you would have ended up with* $y = -8x + 29$.

Problem 17 *Find the area of a right triangle with sides of lengths 5, 3, and 4.*

Answer 17

$$6.$$

Explanation 17 *The longest side here is 5, so it must be the hypotenuse. The area of a triangle is* $\frac{1}{2}bh$ *where* b *is the length of the base and* h *is the height. For a right triangle we can turn it around so that the base is one of the legs, and the height is the other. So we get* $\frac{1}{2} \cdot 3 \cdot 4$.

Problem 18 *Find the area of a circle centered around the point* $(2, 3)$ *so that the point* $(5, 7)$ *is on (the edge of) the circle.*

Answer 18

$$25\pi$$

Explanation 18 *The area of a circle is* πr^2 *where* r *is the radius. To find the radius we find the distance between* $(2, 3)$ *and* $(5, 7)$*, which is given by*

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

which in this case is

$$\sqrt{(5-2)^2 + (7-3)^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

Thus the area is $\pi 5^2 = 25\pi$.